Lecture #5: Introduction to Nondeterministic Finite Automata

Key Concepts

Definition 1. A ***nondeterministic finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

- Q is a finite, nonempty, set of ***states***;
- Σ is an ***alphabet*** such that $Q \cap \Sigma = \emptyset$;
- $\delta : Q \times \Sigma_\lambda \to \mathcal{P}(Q)$ is a ***transition function***;
- $q_0 \in Q$ is the ***start state***; and
- $F \subseteq Q$ is the set of ***accept states***.

Here, $\Sigma_\lambda = \Sigma \cup \{\lambda\}$ and δ is a ***total*** function from $Q \times \Sigma_\lambda$ to $\mathcal{P}(Q)$.

Definition 2. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a nondeterministic finite automaton. The, for $q \in Q$, ***Cl***$_{\boldsymbol{\lambda}}\boldsymbol{(q)}$ is the ***set of states*** that reachable from q by following ***zero or more*** λ-transitions.

Cl$_\lambda(q)$ is sometimes called the ***$\boldsymbol{\lambda}$-closure*** of the state q.

Definition 3. Let $M = (Q, \Sigma, \delta, q_0, F)$. The ***extended transition function*** of M is the total function

$$\delta^\star : Q \times \Sigma^\star \to \mathcal{P}(Q)$$

such that, for $q \in Q$ and $\omega \in \Sigma^\star$,

$$\delta^\star(q, \omega) = \begin{cases} \mathit{Cl}_\lambda(q) & \text{if } \omega = \lambda, \\ \displaystyle\bigcup_{r \in \delta^\star(q,\mu)} \left(\bigcup_{s \in \delta(r,\sigma)} \mathit{Cl}_\lambda(s) \right) & \text{if } \omega = \mu \cdot \sigma \text{ for } \mu \in \Sigma^\star \text{ and } \sigma \in \Sigma. \end{cases}$$

Definition 4. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a nondeterministic finite automaton. Then, for every string $\omega \in \Sigma^\star$, M ***accepts*** ω if

$$\delta^\star(q_0, \omega) \cap F \neq \emptyset$$

and M ***rejects*** ω otherwise.

Definition 5. Let $M = (Q, \Sigma, \delta, q_0, F)$. Then the ***language*** of M, $\boldsymbol{L(M)}$, is the set of strings $\omega \in \Sigma^\star$ such that M accepts ω.