Lecture #5: Introduction to Nondeterministic Finite Automata

Lecture Presentation

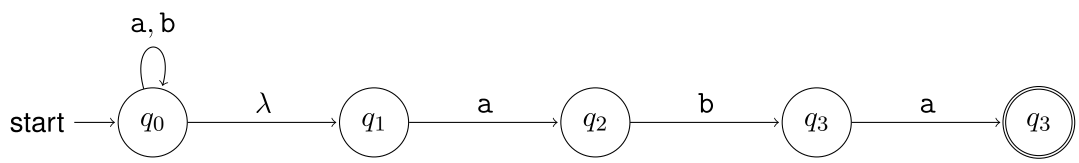
Main Points

A Language and a Nondeterministic Finite Automaton

Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and let $L \subseteq \Sigma^\star$ be the following language:

$$L = \{w \in \Sigma^\star \mid \omega \text{ ends with } \mathtt{aba}\}.$$

Consider the following ***nondeterministic*** finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ with the above alphabet Σ and the following transition diagram.



Problems To Be Solved

1. State the ***type*** of the transition function, δ, that is included as a part of this nondeterministic finite automaton.

   Solution:

2. Give a ***transition table*** for this transition function, δ.

   Solution:

3. The lecture notes introduced ***trees of possibilities*** that showed possible executions of a given nondeterministic finite automaton on a given input string. Draw the tree of possibilities showing the possible executions of this nondeterministic finite automaton on the input string `aaba`.

   Solution:

4. Compute $Cl_\lambda(q)$ for every state, q, in this nondeterministic finite automaton.

   Solution:

5. Use the properties of the extended transition function, given in the lecture notes, to compute $\delta^\star(q_0, \texttt{aaba})$.

   Solution:

6. Use set-theoretic notation (if needed) to describe each of these sets, as precisely you can.

   (a) The set, S_0, of strings in $\Sigma^\star$ whose processing can end in state q_0, that is, the set

   $$\{\omega \in \Sigma^\star \mid q_0 \in \delta^\star(q_0, \omega)\}$$

   Solution:

   (b) The set, S_1, of strings in $\Sigma^\star$ whose processing can end in state q_1, that is, the set

   $$\{\omega \in \Sigma^\star \mid q_1 \in \delta^\star(q_0, \omega)\}$$

   Solution:

   (c) The set, S_2, of strings in $\Sigma^\star$ whose processing can end in state q_2, that is, the set

   $$\{\omega \in \Sigma^\star \mid q_2 \in \delta^\star(q_0, \omega)\}$$

   Solution:

(d) The set, S_3, of strings in $\Sigma^\star$ whose processing can end in state q_3, that is, the set

$$\{\omega \in \Sigma^\star \mid q_3 \in \delta^\star(q_0, \omega)\}$$

Solution:

(e) The set, S_4, of strings in $\Sigma^\star$ whose processing can end in state q_4, that is, the set

$$\{\omega \in \Sigma^\star \mid q_4 \in \delta^\star(q_0, \omega)\}$$

Solution:

7. Describe a ***proof technique***, that you learned about in a prerequisite for this course, that could be used to *prove* that S_0 is the set you claimed, when answering part (a) of the previous question.

   Solution:

8. Describe a ***proof technique***, that you learned about in a prerequisite for this course, that could be used to *prove* that S_0 is the set you claimed, when answering part (a) of the previous question.

   Solution:

Simpler arguments could be given to prove the correctness of your claims about S_1, S_2, S_3, and S_4 after that. Why?

Answer:

9. Assuming that your answers for the above questions are correct, explain why it now follows that the language of this nondeterministic finite automaton is the set of strings in $\Sigma^\star$ that end with `aba`.

   Solution: