Lecture #1: Welcome to CPSC 351!
What Will Happen During the Lecture

Review

The presentation will begin with a ***brief*** review of the material in the preparatory videos and documents for this lecture — and students will have the chance to ask questions about this.

A Problem To Be Solved

Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Suppose that $L_0, L_1, L_2 \subseteq \Sigma^\star$, and that these languages satisfy the following properties.

(a) $\lambda \in L_0$, $\lambda \notin L_1$, and $\lambda \notin L_2$.

(b) For every string $\omega \in \Sigma^\star$ such that $\omega \in L_0$, $\omega \cdot \mathtt{a} \in L_1$, $\omega \cdot \mathtt{a} \notin L_0$, and $\omega \cdot \mathtt{a} \notin L_2$.

(c) For every string $\omega \in \Sigma^\star$ such that $\omega \in L_0$, $\omega \cdot \mathtt{b} \in L_0$, $\omega \cdot \mathtt{b} \notin L_1$, and $\omega \cdot \mathtt{b} \notin L_2$.

(d) For every string $\omega \in \Sigma^\star$ such that $\omega \in L_1$, $\omega \cdot \mathtt{a} \in L_2$, $\omega \cdot \mathtt{a} \notin L_0$, and $\omega \cdot \mathtt{a} \notin L_1$.

(e) For every string $\omega \in \Sigma^\star$ such that $\omega \in L_1$, $\omega \cdot \mathtt{b} \in L_1$, $\omega \cdot \mathtt{b} \notin L_0$, and $\omega \cdot \mathtt{b} \notin L_2$.

(f) For every string $\omega \in \Sigma^\star$ such that $\omega \in L_2$, $\omega \cdot \mathtt{a} \in L_0$, $\omega \cdot \mathtt{a} \notin L_1$, and $\omega \cdot \mathtt{a} \notin L_2$.

(g) For every string $\omega \in \Sigma^\star$ such that $\omega \in L_2$, $\omega \cdot \mathtt{b} \in L_2$, $\omega \cdot \mathtt{b} \notin L_0$, and $\omega \cdot \mathtt{b} \notin L_1$.

Two claims about these languages will be considered.[1]

Claim 1. *Let Σ and L_0 be as given above. Then, for every string $\omega \in \Sigma^\star$, $\omega \in L_0$ if and only if the number of copies of "$\mathtt{a}$" in ω is divisible by 3.*

[1] It is assumed, now, that *modular arithmetic* and *congruences* were introduced in a prerequisite course and that you remember these. Please ask the instructor about these things, if this is not the case.

Claim 2. *Let Σ, L_0, L_1 and L_2 be as given above. Then the following properties are satisfied.*

(a) For every string $\omega \in \Sigma^\star$, $\omega \in L_0$ if and only if the number of copies of "a" in ω is divisible by 3. In other words, *$\omega \in L_0$ if and only if the number of copies of "a" in ω is congruent to 0 modulo 3.*

(b) For every string $\omega \in \Sigma^\star$, $\omega \in L_1$ if and only if the number of copies of "a" in ω is congruent to 1 modulo 3.

(c) For every string $\omega \in \Sigma^\star$, $\omega \in L_2$ if and only if the number of copies of "a" in ω is congruent to 2 modulo 3.

Note that the first claim must be true if the second is — but it is not clear that either one is true, and it might also be possible (at least, in principle) that the first claim is true and the second is false.

The lecture presentation will concern ***proving these claims***.

Things To Do Ahead of Time:

- Prepare questions that you plan to ask about the preparatory videos and supplemental reading — especially if "review material" is not familiar to you.

- Try to prove each of these claims using a proof technique that you learned in a prerequisite course and that has been reviewed in this one. (You might need to try more than one!). What, if anything, can be noticed when you do this?

Other Things That Might Be Discussed

It is not always clear whether a proof is complete and correct. It is also not always clear whether an answer for a question is sufficient, as it is, or needs to be improved.

Some students report that they are overwhelmed by the amount of technical material that is introduced in a course like this one.

If time permits several things that students can do, concerning these things, will be discussed.

Something That will Be Provided Later

One of the above claims *can* be proved using a proof technique that students already know about. It is trickier to prove the other.

A proof of the claim that *can* be proved, along with an explanation why it is *not* very easy to prove the other claim, will be supplied after the lecture presentation.