CPSC 351 — Tutorial Exercise #16
Additional Practice Problems

These problems will not be discussed during the tutorial, and solutions for these problems will not be made available. They can be used as “practice” problems that can help you practice skills considered in the lecture presentation for Lectures #20, or in Tutorial Exercise #16.

Consider the “binary search tree” experiment introduced in the additional practice problems for Tutorial Exercise #15 — which concerned binary search trees storing thee values, namely, the integers 1, 2 and 3.

If this is generalized — so that the number of values to be stored, 3, is replaced by an arbitrary positive integer n, then a binary search tree, storing the set of integers 1 and n (inclusive), is produced these integers into an initially empty tree, in some order. This experiment has a sample space, Ω_n, of size $n!$, whose outcomes each correspond to an order in which the integers can be inserted, in order to construct the tree

As in the previous exercise et us assume that the insertion orders are equally likely, so that the probability distribution is the ***uniform*** probability distribution P : $\Omega_n \to \mathbb{R}$ such that

$$\mathsf{P}(\vec{\alpha}) = \frac{1}{|\Omega_n|} = \frac{1}{n!}$$

for every outcome $\vec{\alpha} \in \Omega_n$.

The ***depth*** of a binary search tree is the maximum number of edges between the root of any tree and any leaf.

1. Compute the expected value of the depth of a binary search tree storing the integers $1, 2, \dots, n$ when $n = 3$.

2. Compute the expected value of the depth of a binary search tree storing the integers $1, 2, \dots, n$ when $n = 4$.