CPSC 351 — Tutorial Exercise #15

Introduction to Discrete Probability Theory for Computer Science

These questions are intended to give you more practices using material about sample spaces, probability distributions, conditional probabilities and independence discussed in Lectures #18 and #19. While almost all of it should be review, some students to not remember this material very well — and some students might not remember seeing it, at all.

Problems To Be Solved

1. Alice and Bob are taking turns doing summersaults. Every time they try, they can either ***succeed*** (that is, complete this move correctly) or ***fail***. They take two turns each — Alice tries; then Bob tries; then Alice tries again; and then Bob tries again.

   (a) Describe how to model this situation using a simple ***sample space*** — which should resemble at least one of the sample spaces introduced in Lecture #18.

   Suppose that Alice and Bob are equally skilled, and they don't get very tired or hurt themselves during this exercise. They also don't get very encouraged or discouraged about past attempts — so that, every time each one of them attempts a summersault, they are just as likely to succeed as they are to fail.

   (b) Give a ***probability distribution***, for your sample space, which models these assumptions.

   When they start, they each have two pieces of candy. Every time that Alice succeeds, Bob gives Alice a piece of candy. Every time that Bob succeeds, Alice gives Bob a piece of candy. No candy is exchanged when someone fails. There are no ways to get more candy and no one eats any candy as this is happening.

   (c) How many pieces of candy might Alice have, when they are done? (List all possibilities).

(d) How many pieces of candy might Bob have, when they are done? What is the relationship between the number of pieces of candy that Alice has, and the number of pieces of candy that Bob has?

(e) What is the probability that Alice has ***strictly more*** pieces of candy than Bob has, when they are done?

2. Suppose that you subscribe to a news service.

   - 40% of the news items come from a *Canadian newspaper*.[1]
   - 25% of the news items come from a *newspaper from the United States.*
   - 20% of the news items come from a *Canadian web site.*
   - 15% of the news items come from a *web site from the United States.*

   It has been observed that 2% of the stories from the Canadian newspaper are conspiracy theories. 10% of the stories from the U.S. newspaper are conspiracy theories. 60% of the stories from the Canadian web site are conspiracy theories, and 90% of the stories from the U.S. web site are conspiracy theories.

   (a) Describe a ***sample space***, Ω, that could be used to model this situation. (The instructor's solution is a set with size eight.)

   (b) State the ***probability distribution*** $\mathrm{P} : \Omega \to \mathbb{R}$ that corresponds to the above information.

   (c) Calculate the conditional probability that a news item came from a web site, given that it is a conspiracy theory.

   (d) Calculate the conditional probability that a news item is a conspiracy theory, given that it came from the United States.

   (e) Is the event that a news item is a conspiracy theory ***attracted to*** the the event that it comes from the United States? If this is not the case then what *can* be said about the relationship between these events, based on the above?

[1] The names of this and other information sources are being withheld, in order to avoid getting the instructor into trouble. ***These statistics have been made up — they probably have no basis in reality, at all.*** Furthermore, this question has been included in a tutorial exercise *before* November, 2024.