Lecture #20: Random Variables and Expectation Questions for Review

Random Variables

Let Ω be a sample space.

1. What is a ***random variable*** over Ω?
2. What are random variables used to model (or "represent")?
3. What is an ***integer-valued random variable*** (over Ω)?
4. What is an ***indicator random variable*** (over Ω)?

Expectation

Once again, let Ω be a sample space — and let $\mathsf{P} : \Omega \to \mathbb{R}$ be a probability distribution for Ω.

5. Let X be a random variable over Ω. What is the ***expected value*** of X with respect to the probability distribution P?
6. Let X be a random variable over Ω, and let $B \subseteq \Omega$ be an event such that $\mathsf{P}(B) > 0$. What is the ***conditional expectation*** of X given B?
7. Let X be a random variable over Ω, and let $B \subseteq \Omega$ be an event such that $\mathsf{P}(B) > 0$ and $\mathsf{P}(B^C) > 0$. How can the expected value of X be computed from the conditional expectations $\mathsf{E}[X \mid B]$ and $\mathsf{E}[X \mid B^C]$, and the probability $\mathsf{P}(B)$ of B?

8. Let X be a random variable over Ω and, for a positive integer n, let $X_1, X_2, \ldots, X_n$ be random variables over Ω such that

$$X = X_1 + X_2 + \cdots + X_n.$$

   (a) How are the expected values of X, and of $X_1, X_2, \ldots, X_n$, related?

   (b) For real numbers c and b, what is the relationship between the expected values of the random variables $cX + b$ and X?

   (c) Is generally true that $\mathsf{E}[X \times Y] = \mathsf{E}[X] \times \mathsf{E}[Y]$, for random variables X and Y over Ω?

9. If X and Y are random variables over Ω, what does it mean for these random variables to be ***independent random variables*** (with respect to a probability distribution P)?

10. If $X_1, X_2, \ldots, X_n$ are random variables over Ω, for a positive integer n, what does it mean for these to be ***pairwise independent random variables*** (with respect to a probability distribution P)?

11. If $X_1, X_2, \ldots, X_n$ are random variables over Ω, for a positive integer n, what does it mean for these to be ***mutually independent random variables*** (with respect to a probability distribution P)?