Lecture #19: Conditional Probability and Independence
Questions for Review

Conditional Probability

1. Let Ω be a sample space, let P be a probability distribution for Ω, and let $A, B \subseteq \Gamma$ be events such that $\mathsf{P}(B) > 0$. What is the ***conditional probability***, $\mathsf{P}(A \,|\, B)$, of A given B?
2. Let Ω, P and B be as above. What is the ***conditional probability space (defined from P) conditional on event B***?
3. State the ***Law of Total Probability***. What might it be used for?
4. State the ***Extended Partition Rule***. How is it related to the "Law of Total Probability"? What might it be used for?
5. State ***Baye's Theorem***. How might *this* be used?

Independence

6. Consider a sample space Ω, probability distribution $\mathsf{P} : \Omega \to \mathbb{R}$, and a pair of events $A, B \subseteq \Omega$ such that $\mathsf{P}(B) > 0$.
   (a) What does it mean for A to be ***attracted*** to B (under P)?
   (b) What does it mean for A to be ***repelled*** by B (under P)?
   (c) What does it mean for A to be ***indifferent*** to B (under P)?
7. What does it mean for a pair of events, $A \subseteq \Omega$ and $B \subseteq \Omega$, to be ***independent***?
8. Consider a sample space Ω and events $A \subseteq \Omega$ and $B_1, B_2, \ldots, B_k \subseteq \Omega$, for an integer $k \geq 1$. Let $\mathsf{P} : \Omega \to \mathbb{R}$ be a probability distribution for the sample space Ω.
   (a) What does it mean for the events $B_1, B_2, \ldots, B_k$ to be ***pairwise disjoint***?

Now suppose that $B_1, B_2, \dots, B_k$ are pairwise disjoint and, furthermore, that

$$B_1 \cup B_2 \cup \cdots \cup B_k = \Omega.$$

(b) Describe how the probability, $\mathsf{P}(A)$, of the event A can be calculated from the conditional probabilities $\mathsf{P}(A \mid B_i)$, for $1 \leq i \leq k$.

(c) What can be concluded about $\mathsf{P}(A)$, and the relationship between A and B_i, if

$$\mathsf{P}(A \mid B_1) = \mathsf{P}(A \mid B_2) = \cdots = \mathsf{P}(A \mid B_k) = q$$

for some real number q?

9. Once again, let Ω be a sample space, let $\mathsf{P} : \Omega \to \mathbb{R}$ be a probability distribution for the sample space Ω, and let $A_1, A_2, \dots, A_k \subseteq \Omega$ be events, for an integer k such that $k \geq 2$.

   (a) What does it mean for the events $A_1, A_2, \dots, A_k$ to be ***mutually independent***?

   (b) What does it mean for the events $A_1, A_2, \dots, A_k$ to be ***pairwise independent***?

   (c) How are "mutual independence" and "pairwise independence" related?