CPSC 351 — Tutorial Exercise #13

Additional Practice Problem

This problem will not be discussed during the tutorial, and solutions for this problem will not be made available. It can be used as a "practice" problem that can help you practice skills considered in the lecture presentation for Lectures #15–17, or in Tutorial Exercise #13.

Consider an alphabet $\Sigma_{\text{2TM}} = \Sigma_{\text{TM}} \cup \{\#\}$. Recall, from the lecture presentation for Lecture #17, that a *pair* of Turing machines M_1 and M_2 can be encoded as a string $\alpha\#\beta \in \Sigma^\star_{\text{2TM}}$ where $\alpha \in \text{TM} \subseteq \Sigma^\star_{\text{TM}}$ is the encoding for M_1 and $\beta \in \text{TM} \subseteq \Sigma^\star_{\text{TM}}$ is the encoding for M_2.

As in the presentation for Lecture #17, let $\text{Pair}_{\text{TM}} \subseteq \Sigma^\star_{\text{2TM}}$ be the language of encodings of pairs of Turing machines

$$M_1 = (Q_1, \Sigma, \Gamma_1, \delta_1, q_{0,1}, q_{A,1}, q_{R,1})$$

and

$$M_2 = (Q_2, \Sigma, \Gamma_2, \delta_2, q_{0,2}, q_{A,2}, q_{R,2})$$

with the same input alphabet Σ. It was proved, during the presentation for Lecture #17, that the language Pair_{TM} is ***decidable***.

1. Let

   $$\text{Subset}_{\text{TM}} \subseteq \text{Pair}_{\text{TM}} \subseteq \Sigma^\star_{\text{2TM}}$$

   be the language including encodings of pairs of Turing machines M_1 and M_2, with the same input alphabet Σ, such that $L(M_1) \subseteq L(M_2)$. Prove that the language $\text{Subset}_{\text{TM}}$ is ***undecidable***.