

# Lecture #17: Proofs of Undecidability — Examples II

## Lecture Presentation

### Goal for This Lecture

Let  $\Sigma_{2\text{TM}} = \Sigma_{\text{TM}} \cup \{\#\}$ . A pair of Turing machines  $M_1$  and  $M_2$  can be encoded as a string  $\alpha\#\beta \in \Sigma_{2\text{TM}}^*$  where  $\alpha \in \text{TM} \subseteq \Sigma_{\text{TM}}^*$  is the encoding for  $M_1$  and  $\beta \in \text{TM} \subseteq \Sigma_{\text{TM}}^*$  is the encoding for  $M_2$ .

1. Let  $\text{Pair}_{\text{TM}} \subseteq \Sigma_{2\text{TM}}^*$  be the language of encodings of pairs of Turing machines

$$M_1 = (Q_1, \Sigma, \Gamma_1, \delta_1, q_{0,1}, q_{A,1}, q_{R,1})$$

and

$$M_2 = (Q_2, \Sigma, \Gamma_2, \delta_2, q_{0,2}, q_{A,2}, q_{R,2})$$

**with the same input alphabet  $\Sigma$ .**

**Goal #1:** Prove that the language  $\text{Pair}_{\text{TM}}$  is **decidable**.

2. Now let

$$E_{\text{TM}} \subseteq \text{Pair}_{\text{TM}} \subseteq \Sigma_{2\text{TM}}^*$$

be the language including encodings of pairs of Turing machines  $M_1$  and  $M_2$ , with the same input alphabet  $\Sigma$ , such that  $L(M_1) = L(M_2)$ .

**Goal #2:** Prove that the language  $E_{\text{TM}}$  is **undecidable**.

## **Proving That $\text{Pair}_{\text{TM}}$ is Decidable**

### **Useful Properties**

### **A “High-Level” Algorithm**

**Implementation-Level Details**

## **Proving That $E_{TM}$ is Undecidable**

**Undecidable Languages That We Already Know About**

**Which Many-One Reduction Will We Try To Establish?**

## **Useful Aspects of The Problems of Interest — and How To Use Them**

## Using the Decidability of a Related Language

**Thinking about Turing Machines and Input Strings: What Should Our Mapping Be?**

**Describing This in More Detail (if Needed)**



## Specifying a Suitable Function $f$

## **A First Claim about This Function**

## **A Second Claim about This Function**

**A Third Claim about This Function**

**A Useful Related Result, That You Might Establish First**

## The Third Result and Its Proof

**Finishing Up**

**Something Helpful To Remember About This Problem**