Lecture #14: Oracle Reductions Lecture Presentation

A First Problem — Suitable for a Test

Let $\Sigma = \{a, b, c\}$, let $L \subseteq \Sigma^{\star}$, and let

 $L_a = \{ \omega \in \Sigma^{\star} \mid \omega \in L \text{ and } \omega \text{ ends with "a"} \}.$

You were asked to prove that $L_a \preceq_{O} L$.

What Do You Need To Provide?

Starting with an (Incomplete) Algorithm

Establishing Correcteness

Adding Implementation Details — and Completing the Proof

A Second Problem — More Suitable for an Assignment

Consider the language LOOP_{TM} $\subseteq \Sigma_{TM}^*$, including encodings of Turing machines M and input strings ω for M such that M **loops** on ω .

You were asked to establish an oracle reduction, involving $LOOP_{TM}$ and some other language, that can be used to prove that the language $LOOP_{TM}$ is undecidable.

Similar Languages

A t this point in the course several similar languages have been considered:

- The language TM+I $\subseteq \Sigma^*_{TM}$ of encodings of Turing machines M and input strings ω for M. This language is **decidable** and it follows from the definitions of these languages that LOOP_{TM} $\subseteq \Sigma_{TM}$.
- The language A_{TM} ⊆ Σ^{*}_{TM} of encodings of Turing machines M and input strings ω for M such that M accepts ω.

This language is *recognizable*: A multi-tape Turing machine with language A_{TM} (called a "universal Turing machine") was described in Lecture #12 — and it follows, by results about multi-tape Turing machines included in Lecture #10, that there must also exist a standard (single tape) Turing machine, $M_{A_{TM}}$, whose language is A_{TM} , as well.

On the other hand it was proved in Lecture #13, that the language A_{TM} is *undecidable*.

The goal of this lecture presentation will be to use an *oracle reduction* — along with the above information — to prove that the language LOOP_{TM} is also *undecidable*.

Which Reduction Should We Use? Why?

An Algorithm That Uses a Subroutine

Establishing Correctness

Adding Implementation-Level Details

How One Would Finish (If We Had Time and Wanted To Do Everything