CPSC 351 — Tutorial Exercise #11

Variants of Turing Machines — and Simulations

This exercises is intended to help you to understand how ***simulations*** can be used to show that additional "variants" of Turing machines are equivalent — that is, they define the same sets of "Turing-recognizable" and "Turing-decidable" languages, as well as "Turing-computable" functions.

Problems To Be Solved

1. A ***Turing machine whose tape head can stay*** is exactly the same as a standard Turing machine, as defined in the first lecture on Turing machines, except the transition function is a function

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{\mathtt{L}, \mathtt{R}, \mathtt{S}\}$$

so that the tape head is also allowed to stay where it is.

(a) Show that if a language $L \subseteq \Sigma^\star$ is recognized by a standard Turing machine M then L is also recognized by some Turing machine whose tape head can stay.

(b) Show that if a language $L \subseteq \Sigma^\star$ is recognized by a Turing machine whose tape head can stay then L is also recognized by some standard Turing machine,

(c) Show that if a language $L \subseteq \Sigma^\star$ is decided by a standard Turing machine M then L is also decided by some Turing machine whose tape head can stay.

(d) Show that if a language $L \subseteq \Sigma^\star$ is decided by a Turing machine whose tape head can stay then L is also decided by some standard Turing machine,

It follows that the sets of *Turing-recognizable* and *Turing-decidable* languages would not be changed if they were defined using Turing machines whose tape heads can stay instead of standard Turing machines.

Note: You should find that some parts of this problem are *extremely* easy to solve and that other parts are almost as easy.

2. A ***Turing machine with a doubly infinite tape*** is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right.

   The tape is initially filled with blanks except for the portion that contains the input — and the tape head is initially pointing to the leftmost symbol in the input if the input is nonempty.

   Computation is as usual, except that the tape head never encounters an end of the tape as it moves left.

   Describe a ***simulation*** of a Turing machine with a doubly infinite tape, by a standard Turing machine, that can be used to prove the following: For every language $L \subseteq \Sigma^\star$, over every alphabet Σ, the following is true.

   (a) If there is a Turing machine with a doubly infinite tape that recognizes L then L is Turing-recognizable.

   (b) If there is a Turing machine with a doubly infinite tape that decides L then L is Turing-decidable.

3. Describe another ***simulation*** — this time, of a standard Turing machine, by a Turing machine with a doubly infinite tape, that can be used to prove the following as well.

   (a) If L is Turing-recognizable then there is a Turing machine with a doubly infinite tape that recognizes L.

   (b) If L is Turing-decidable then there is a Turing machine with a doubly infinite tape that decides L.

Note: It should not be hard to see, in each of these cases, that the new kind of Turing machine, being considered, would also be used to define the same set of ***computable functions*** as before — provided that, for Turing machines with two-way infinite tapes, we require the output to be written on the tape (surrounded by blanks), on termination, with the tape head pointed to the cell containing the leftmost symbol in the output (whenever this is not the empty string).