CPSC 351 — Tutorial Exercise #10
Additional Practice Problems

About These Problems

These problems will not be discussed during the tutorial, and solutions for these problems will not be made available. They can be used as "practice" problems that can help you practice skills considered in the lecture presentation for Lecture #11, or in Tutorial Exercise #10.

Practice Problems

If you completed (or made a good start on) the problems in Tutorial Exercise #10, and you want to see additional evidence that Turing machines are — in some sense — "as powerful as real computers", then the following additional properties might be considered. In these questions, the alphabet Σ_p and the language $L_{\textsf{pair}} \subseteq \Sigma_p^\star$ are as defined in the tutorial exercise; $\Sigma_b = \{0, 1\}$.

1. *Comparison*: Design a multi-tape Turing machine that decides the language $L_\leq \subseteq \Sigma_p^\star$, consisting of all strings $\omega = \mu\#\nu \in L_{\textsf{pair}}$ such that μ represents a nonnegative integer n, and ν represents a nonnegative integer m, where $n \leq m$.

2. *Quotient:* Design a multi-tape Turing machine that computes a function $f_{\textsf{quo}} : \Sigma_p^\star \to \Sigma_b^\star$ which satisfies the following properties:

   (i) If $\omega \in \Sigma_p^\star$ such that $\omega \notin L_{\textsf{pair}}$ then $f_{\textsf{quo}}(\omega) = \lambda$.

   (ii) If $\omega \in L_{\textsf{pair}}$, so that $\omega = \mu\#\nu$ for $\mu, \nu \in \Sigma_b^\star$, and $\nu = 0$ (so that ν represents zero) then $f_{\textsf{quo}}(\omega) = \lambda$.

   (iii) In the only other case $\omega = \mu\#\nu$, for $\mu, \nu \in \Sigma_b^\star$, where μ and ν represent nonnegative integers n and m, respectively — and, furthermore, $m \geq 1$. In this case $f_{\textsf{quo}}(\omega)$ is the unpadded binary representation of $\lfloor n/m \rfloor$, that is, the *quotient* obtained when applying integer division with remainder, to divide n by m.

3. *Remainder:* Design a multi-tape Turing machine that computes a function $f_{\textsf{rem}} : \Sigma_p^\star \to \Sigma_b^\star$ that is also equal to λ in cases (i) and (ii), above, and such that $f_{\textsf{rem}}(\omega)$ is the unpadded binary representation of the *remainder*, $n \bmod m$, obtained by dividing n by m, in case (iii).