

CPSC 351 — Tutorial Exercise #9

Additional Practice Problems

About These Problems

These problems will not be discussed during the tutorial, and solutions for these problems will not be made available. They can be used as “practice” problems that can help you practice skills considered in the lecture presentation for Lecture #10, or in Tutorial Exercise #9.

Practice Problems

Let

$$\Sigma_D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

and let $L_{\text{dec}} \subseteq \Sigma_D^*$ be the set of unpadding *decimal* representations of the natural numbers — so that L_D is the union of the set $\{0\}$ and the set of all nonempty strings in Σ_D^* that do not begin with “0”. Consider the following functions $\widehat{f}_{+1}, \widehat{f}_{-1} : \Sigma_D^* \rightarrow \Sigma_D^*$ that are defined as follows.

- If $\omega \in L_{\text{dec}}$ and, furthermore, ω is the unpadding decimal representation of some *positive* integer n then $\widehat{f}_{+1}(\omega)$ is the unpadding decimal representation of $n + 1$, and \widehat{f}_{-1} is the unpadding decimal representation of $n - 1$.
- If $\omega = 0$, so that ω is the unpadding decimal representation of 0, then $\widehat{f}_{+1}(\omega) = 1$, the unpadding decimal representation of 1, and $\widehat{f}_{-1}(\omega) = \lambda$, the empty string.
- If $\omega \in \Sigma_D^*$ but $\omega \notin L_{\text{dec}}$ then $\widehat{f}_{+1}(\omega) = \widehat{f}_{-1}(\omega) = \lambda$, the empty string.

Recall that algorithms for corresponding problems in binary arithmetic and decimal arithmetic are often similar.

1. Modify information from the lecture presentation for Lecture #10 (concerning a Turing machine to compute the function f_{+1}) to produce a high-level description and implementation-level description of Turing machine that computes the function \widehat{f}_{+1} .
2. Modify information from Tutorial Exercise #10 and its solutions (concerning a Turing machine to compute the function f_{-1}) to produce a high-level description and implementation-level description of a Turing machine that computes the function \widehat{f}_{-1} .