CPSC 351 — Tutorial Exercise #8
Additional Practice Problems

About These Problems

These problems will not be discussed during the tutorial, and solutions for these problems will not be made available. They can be used as "practice" problems that can help you practice skills considered in the lecture presentation for Lectures #8 and #9, or in Tutorial Exercise #8.

Practice Problems

1. Let $\Sigma = \{\mathtt{a}\}$. Prove that the language

   $$L = \{a^n \mid n \text{ is a power of two}\}$$

   is not a regular language — where "n is a power of two" means that $n = 2^k$ for some integer k such that $k \geq 0$.

2. Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Prove that the language

   $$L = \{\mathtt{ba}^n\mathtt{ba}^{2n}\mathtt{b} \mid n \text{ is an integer such that } n \geq 0\}$$

   is not a regular language.

3. Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Prove that the language

   $$L = \{\mathtt{a}^k\mathtt{ba}^\ell\mathtt{ba}^{k+\ell} \mid k \text{ and } \ell \text{ are integers such that } k, \ell \geq 0\}$$

   is not a regular language.

The next problem is somewhat different from the other problems being considered, and it is OK if you do not know how to solve it.

4. Suppose that Σ and $\widehat{\Sigma}$ are alphabets such that $\Sigma \subset \widehat{\Sigma}$, and suppose that $L \subseteq \Sigma^\star$ such that L is not a regular language.

   Now let $\widehat{L} \subseteq \Sigma^\star$ such that

   $$\widehat{L} = \{\omega \in \widehat{\Sigma}^\star \mid \omega \in L \subseteq \Sigma^\star\}$$

   — so that L and $\widehat{L}$ are the same *sets* (of strings) — but these are considered to be languages over different alphabets.

   Prove that $\widehat{L}$ is *also* not a regular language.

You may use the result, proved when solving the above problems, when solving the problems after this one. It ***should not*** be necessary to use the Pumping Lemma for Regular Languages — but the languages have been proved not to be regular, using this result, might be useful here.

5. Let $\widehat{\Sigma} = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. Use one or more ***closure properties*** to to prove that the language

   $$L = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^m \mid n, m \geq 0\}$$

   is not a regular language.

6. Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, let

   $$L = \{\mathtt{a}^k\mathtt{b}\mathtt{a}^\ell\mathtt{b}\mathtt{a}^{k+\ell} \mid k, \ell \geq 0\} \subseteq \Sigma^\star,$$

   and let

   $$\widehat{L} = \{\mathtt{a}^k\mathtt{b}\mathtt{a}^\ell\mathtt{b}\mathtt{a}^m \mid k, \ell, m \geq 0 \text{ and } m \neq k + \ell\} \subseteq \Sigma^\star.$$

   Use one or more ***closure properties*** to prove that if L is not a regular language then $\widehat{L}$ is not a regular language, either.

   Note: $\widehat{L}$ is ***not*** the complement of L.

7. At this point, you might be imagining lots of *other* "closure properties" for the set of regular languages. With that noted, consider the following.

   > ***Claim:*** Let Σ be an alphabet. Then, for all languages L_1 and L_2 such that $L_1, L_2 \subseteq \Sigma^\star$ and $L_1 \subseteq L_2$, if L_2 is a regular language then L_1 is a regular language too.

   (a) Is this claim true?

(b) Say, as precisely as you can, how you could prove your answer for (a) if you said that this claim is ***true***.

(c) Say, as precisely as you can, how you could prove your answer for (a) if you said that this claim is ***false***.

(d) Prove your answer for (a).

(e) What, if anything, can you conclude about a language $L_1 \subseteq \Sigma^\star$, for an alphabet Σ, if there exists a language $L_2 \subseteq \Sigma^\star$ such that $L_1 \subseteq L_2$ and L_2 is regular?

8. Say whether each of the following possible "closure properties" is correct and then prove that your answer is correct.

(a) *Possible Closure Property:* Let $L_1, L_2 \subseteq \Sigma^\star$. If L is a regular language, and $L_1 \cap L_2$ contains infinitely many strings in $\Sigma^\star$, then L_2 is a regular language.

(b) *Possible Closure Property:* Let $L_1, L_2 \subseteq \Sigma^\star$. If L_1 and L_2 are regular languages then the language

$$L_1 \oplus L_2 = \{\omega \in \Sigma^\star \mid \omega \text{ belongs to } \textit{exactly one} \text{ of } L_1 \text{ or } L_2\} \subseteq \Sigma^\star$$

is a regular language.