

Lecture #8: Nonregular Languages, Part One

Lecture Presentation

This lecture introduces the following result.

Pumping Lemma: Let Σ be an alphabet and let $A \subseteq \Sigma^*$.

If A is a regular language, then there is a number $p \geq 1$ (called the **pumping length** for A) — which only depends on A — such that if s is any string in A with length at least p , then s can be divided into three pieces $s = xyz$ (for $x, y, z \in \Sigma^*$), satisfying the following three conditions.

1. $xy^iz \in A$ for every integer i such that $i \geq 0$.
2. $|y| > 0$ (so that $y \neq \lambda$).
3. $|xy| \leq p$.

Another to write this — so that the “quantification of variables” is clearer:

This result will be used, during this lecture presentation, to show that if $\Sigma = \{a\}$ then the language

$$L = \{a^{(n^2)} \mid n \in \mathbb{N}\} \subseteq \Sigma^*$$

is not regular.

Getting Started

What ***proof technique*** should be used, when the “Pumping Lemma for Regular Languages” is to be applied to prove that a language is not regular?

Why is it often a good idea to write the “Pumping Lemma for Regular Languages” out, near the beginning of a proof where you are using it?

What gets introduced (in the argument) right away? What, if anything, can be assumed about this? Why?

The **second** thing that gets introduced, is a string s . What properties do you need to show that this string has? Why?

List some reasonable choices for the “string s ” that you might choose, when you are trying use the “Pumping Lemma for Regular Languages” to prove that the above language, L , is not regular.

What else must be introduced, and what more do you need to do, to complete the proof?

Note that a goal, here, is to prepare you to use the “Pumping Lemma for Regular Languages” to prove the *other* languages (which you do not know about, right now) are not regular — and that ***you will probably get better at this with practice.***

Try to use one or more of the strings s that you listed, above, and try to finish the proof. Describe what happens. What, if anything, can you learn about making a “good” choice of the string s , if you were not above to finish?¹

¹There will probably not be time to consider very many strings during the lecture presentation — but this is something that you can do, on your own, either before or after the lecture presentation, to have more practice.

Now (perhaps, after the instructor has described it) describe a string s that **can** be used to complete a correct proof that the above language L is not a regular language.

List the points that must be made (or the things that must be done) to finish — explaining *why* the various things, that must be established, are true when you use the above string s .

Something More To Do On Your Own

Use the above to write a proof that the language L is not regular — that uses the above ideas, but that is organized and written in a way that make it as easy as possible for another reader to understand and believe it.