Lecture #6: Equivalence of Deterministic Finite Automata and Nondeterministic Finite Automata

Questions for Review

1. Give a brief proof that if there exists a deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ whose language is $L \subseteq \Sigma^\star$, then there exists a ***nondeterministic*** finite automaton $\widehat{M}$ whose language is L as well.
2. Suppose, instead, that you have been given a ***nondeterministic*** finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ whose language is $L \subseteq \Sigma^\star$ and that you wish to design a ***deterministic*** finite automaton $\widehat{M} = (\widehat{Q}, \Sigma, \widehat{\delta}, \widehat{q}_0, \widehat{F})$ whose language is L as well.
   (a) What *information* must $\widehat{M}$ remember in order to correctly decide whether a given string belongs to L? That is, what *information* needs to be considered and used in order to define the states in $\widehat{Q}$?
   (b) Describe a process that you can follow in order to use M to define both the set $\widehat{Q}$ of states in $\widehat{M}$ and the transition function $\widehat{\delta}$.
   (c) Which state in $\widehat{Q}$ should be the start state $\widehat{q}_0$? Why?
   (d) How should be set $\widehat{F}$ of accepting states in $\widehat{M}$ be defined?
3. Suppose that $|Q| = n$ for a positive integer n. How large might $\widehat{Q}$ be, as a function of n?