Lecture #5: Introduction to Nondeterministic Finite Automata
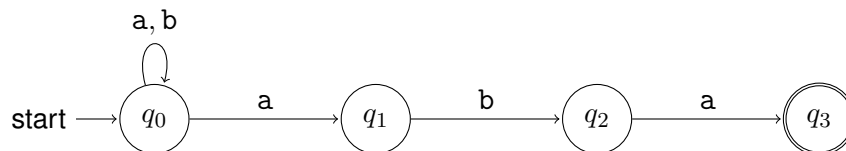
Lecture Presentation

A Language and a Nondeterministic Finite Automaton

Let $\Sigma = \{a, b\}$ and let $L \subseteq \Sigma^\star$ be the following language:

$$L = \{w \in \Sigma^\star \mid \omega \text{ ends with } \mathtt{aba}\}.$$

Consider the following **nondeterministic** finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ with the above alphabet Σ and the following transition diagram.

Problems To Be Solved

1. State the **type** of the transition function, δ, that is included as a part of this nondeterministic finite automaton.

   Solution:

2. Give a **transition table** for this transition function, δ.

   Solution:

3. The lecture notes introduced **trees of possibilities** that showed possible executions of a given nondeterministic finite automaton on a given input string. Draw the tree of possibilities showing the possible executions of this nondeterministic finite automaton on the input string `ababa`.

   Solution:

4. Compute $Cl_\lambda(q)$ for every state, q, in this nondeterministic finite automaton.

   Solution:

5. Use the properties of the extended transition function, given in the lecture notes, to compute $\delta^\star(q_0, \mathtt{ababa})$.

   Solution:

6. Use set-theoretic notation to describe each of these sets, as precisely you can.

(a) The set, S_0, of strings in $\Sigma^\star$ whose processing can end in state q_0, that is, the set

$$\{\omega \in \Sigma^\star \mid q_0 \in \delta^\star(q_0, \omega)\}$$

Solution:

(b) The set, S_1, of strings in $\Sigma^\star$ whose processing can end in state q_1, that is, the set

$$\{\omega \in \Sigma^\star \mid q_1 \in \delta^\star(q_0, \omega)\}$$

Solution:

(c) The set, S_2, of strings in $\Sigma^\star$ whose processing can end in state q_2, that is, the set

$$\{\omega \in \Sigma^\star \mid q_2 \in \delta^\star(q_0, \omega)\}$$

Solution:

(d) The set, S_3, of strings in $\Sigma^\star$ whose processing can end in state q_3, that is, the set

$$\{\omega \in \Sigma^\star \mid q_3 \in \delta^\star(q_0, \omega)\}$$

Solution:

7. Describe a **_proof technique_**, that you learned about in a prerequisite for this course, that could be used to *prove* that S_0 is the set you claimed, when answering part (a) of the previous question.

   Solution:

   Simpler arguments could be given to prove the correctness of your claims about S_1, S_2 and S_3 after that. Why?

   Answer:

8. Assuming that your answers for the above questions are correct, explain why it now follows that the language of this nondeterministic finite automaton is the set of strings in $\Sigma^\star$ that end with `aba`.

   Solution: