CPSC 351 — Tutorial Exercise #1
Additional Practice Problems

About These Problems

These problems will not be discussed during the tutorial, and solutions for these problems will not be made available. They can be used as "practice" problems that can help you practice skills considered in the lecture presentation for Lecture #1, or in Tutorial Exercise #1. Along with the review material needed for Tutorial Exercise #1, these may involve material introduced in the required reading for Lecture #1:

- Lecture #1: Alphabets, Strings, and Languages

Students, who are interested in doing so, may contact the course instructors for comments about their attempts to solve these problems.

Practice Problems

1. Consider a sequence of integers

$$a_0, a_1, a_2, \ldots$$

such that, for every integer n such that $n \geq 0$,

$$a_n = \begin{cases} 0 & \text{if } n = 0, \\ 1 & \text{if } n = 1, \\ 2 \cdot a_{n-1} - a_{n-2} & \text{if } n \geq 2. \end{cases}$$

Note that it follows, from this definition, that

$$\begin{aligned} a_2 &= 2 \cdot a_1 - a_0 \\ &= 2 \cdot 1 - 0 \\ &= 2. \end{aligned}$$

(a) Find a “closed form” expression for a_n, for every integer n such that $n \geq 0$ (this should be a very simple polynomial function of n)

(b) Prove that your answer for part (a) is correct.

2. Let k be a positive integer, and let

$$\Sigma_k = \{\sigma_1, \sigma_2, \ldots, \sigma_k\}$$

be an alphabet with size k. Let M_{Σ_k} be the set of strings in $\Sigma_k^\star$ with the form

$$\alpha_1^{c_1} \alpha_2^{c_2} \ldots \alpha_k^{c_k}$$

such that $c_i \geq 0$ for every integer i such that $1 \leq i \leq k$ — that is, the set of strings in $\Sigma_k^\star$ that do not include a copy of the symbol σ_j *before* any copies of the symbol σ_i, for all integers i and j such that $1 \leq i < j \leq k$.

Now, for $k \geq 1$ and $n \geq 0$, let $s_{n,k}$ be the number of strings in M_{Σ_k} with length n, so that

$$|M_{\Sigma_k} \cap \Sigma_k^n| = s_{n,k}.$$

Prove that

$$s_{n,k} = \binom{n+k-1}{k-1}$$

for all integers k and n such that $n \geq 0$ and $k \geq 1$.