Lecture #1: Introduction to CPSC 513

Lecture Presentation

Consider the following program (written using a simple programming language that will be discussed, later in this course). Here, X is an input variable whose initial value is some non-negative integer, and Z is an integer-valued variable.

```
      Z ← 0
      IF X ≠ 0 GOTO A
      GOTO B
[A]   X ← X − 1
      Z ← Z + 1
      IF X ≠ 0 GOTO A
[B]   EXIT
```

(a) Suppose X has initial value 0. How many times is the step with label A reached and executed during the execution of this program, in this case? What are the values of X and Z when the step with label B is reached, and this execution of the algorithm ends?

(b) Answer the questions in (a) for an execution of the algorithm when X has initial value 1.

Now let n be an integer such that $n \geq 2$.

(c) Suppose X has initial value n. How many times is the step with label A reached and executed during an execution of the algorithm, in this case?

(d) Let ℓ be your answer for (c) (so that ℓ is an integer-valued function of n) and let k be an integer such that $1 \leq k \leq \ell$. What values do the variables X and Z have when the step with label A is reached for the k^{th} time (but before this statement is executed)?

(e) Once again, consider an execution of the algorithm when X has initial value n. What are the values of X and Z when the step with label B is reached, and this execution of the algorithm ends?

Suppose that you do not know the answers to these questions right away — or, possibly, are not even sure what these questions mean. Please describe things that you might do (which might be helpful) to make sure that you *do* understand the questions, and to help you to discover the answers.

The answer for these questions are as follows:

(a)

(b)

(c)

(d)

(e)

Suppose, next, that you are asked to ***prove*** that your answers are correct.

Please list things that you might do to *discover* a proof and confirm that your proof is complete and correct.

Suppose, now, that you have decided that you know how to prove these claims. List things that you should think about, and do, so that your proof is as readable (by another person who has successfully completed the prerequisites for this course but who does not already believe that your claims are true and understand *why*) as possible.

Something To Do Before the Next Lecture: Write each of these proofs yourself, using the information you discovered. If there is time then you will have a chance to compare the proofs that you wrote and see whether you can find ways to improve them.