Asymptotic Notation and Standard Functions

A Suggested Exercise

About This Exercise

This exercise is intended to be an opportunity to learn (and practice) using asymptotic notation to state relationships between the rates of growths of functions and to prove that these relationships hold.

Problems To Be Discussed in This Tutorial

1. (Based on *Introduction to Algorithms*, Problem 3-2)

   For each of the following functions f and g, use asymptotic notation to express the relationships between these functions as precisely as you can. You may assume that k, ϵ and c are constants such that $k \geq 1$, $\epsilon > 0$, and $c > 1$.

   (a) $f(n) = \log_2^k n$ and $g(n) = n^\epsilon$.
   (b) $f(n) = n^k$ and $g(n) = c^n$.
   (c) $f(n) = \sqrt{n}$ and $g(n) = n^{\sin n}$ — assuming here, that (when computing $\sin n$) n is some number of *degrees* rather than *radians*.
   (d) $f(n) = 2^n$ and $g(n) = 2^{n/2}$
   (e) $f(n) = n^{\log_2 c}$ and $g(n) = c^{\log_2 n}$

2. Let $f(n) = 3n^3 + 2n + 1$ and let $g(n) = n^3$.

   (a) Use the *definition* of $O(g)$ to prove that $f \in O(g)$.
   (b) Use a *limit test* to prove that $f \in O(g)$.
   (c) Now that you have computed the limit needed to answer the previous part of this question, consider the other "limit tests" for asymptotic notation. What *else* can concluded, about the relationship between f and g, based on the limit that you have computed?