Asymptotic Notation and Standard Functions

A Sample Assignment

1. Consider the functions $f, g : \mathbb{R} \to \mathbb{R}$ such that, for all $x \in \mathbb{R}$, $f(x) = x \cdot \sqrt{x}$ and $g(x) = x^2$.

   (a) Use the definition of "$o(x^2)$" to prove that $x \cdot \sqrt{x} \in o(x^2)$.

   (b) Use a *limit test* to prove that $x \cdot \sqrt{x} \in o(x^2)$.

2. Let $f, g : \mathbb{R} \to \mathbb{R}$ be asymptotically positive functions. Prove that if $f \in \Omega(g)$ if and only if $g \in O(f)$.