

# Review of Proofs and Mathematical Induction

## A Suggested Exercise

### About This Exercise

Mathematical induction is, almost certainly, the most important proof technique for problems in computer science that students learn about in Mathematics 271 or 273. It is used in CPSC 331 over and over again — and you can expect to be asked to use it in one or more senior computer science courses that you take later on.

Unfortunately it also seems to be the proof technique that many students have most trouble with. This exercise is intended to remind you of this proof technique, give you a bit more practice using it, and to help you to look for some mistakes that students often make when they use mathematical induction, so that you can avoid them.

### Problem To Be Discussed in the “Tutorial”

1. Consider the following **claim**: For every integer  $n$  such that  $n \geq 2$ ,

$$\prod_{i=2}^n \left(1 - \frac{1}{i}\right) = \frac{1}{n}.$$

Suppose that you want to write a **proof** of this claim, using mathematical induction — specifically, induction on  $n$ , using the standard form of mathematical induction.

- (a) Write down the **result** that you need to prove in the **basis**. Please try to be as precise as you can when answering this and other questions.
- (b) Now consider the **inductive step**. This should begin a sentence like the following:

Let  $k$  be an integer such that  $k \geq 2$ .

Write down the **Inductive Hypothesis** that can be assumed during the inductive step. (*Note*: This should have something to do with the case that  $n = k$ .)

- (c) Now write down the **Inductive Claim** that you must *prove* in the inductive step. (*Note*: This should have something to do with the case that  $n = k + 1$ .)
- (d) If you have answered the above questions then you have mapped out the **structure** of a proof of the above claim.

Fill in the details, in order to produce a complete **proof** of the claim using mathematical induction on  $n$ .