Small-world phenomenon

Jeroen Keijser

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Table of Contents

- Introduction
  - Colloquial definition
- Duncan Watts
  - motivation
  - mathematical definition □,□,□,□
- Do they really exist?
  - Kevin Bacon
  - Power Grid
  - Worm
- Dynamic Systems
  - Disease Spreading
-

Table of Contents Cont.

- As a Computational Model
  - cellular automata
  - □ revisited
- Small world after all?
  - Milgram revisited
- Conclusions/Questions

"Colloquial definition"

- You meet someone on the train, at a party, at some show, (perhaps even in another country) and you realize that you have a common acquaintance/friend. In surprise you exclaim, "It's a small world…"

“Six Degrees of Separation”

- The common belief that between any one person(say a Calgary Computer Scientist) and any other one person (say Saddam Hussein) there are at most only 6 links… (a friend of brother of spouse of …etc.)

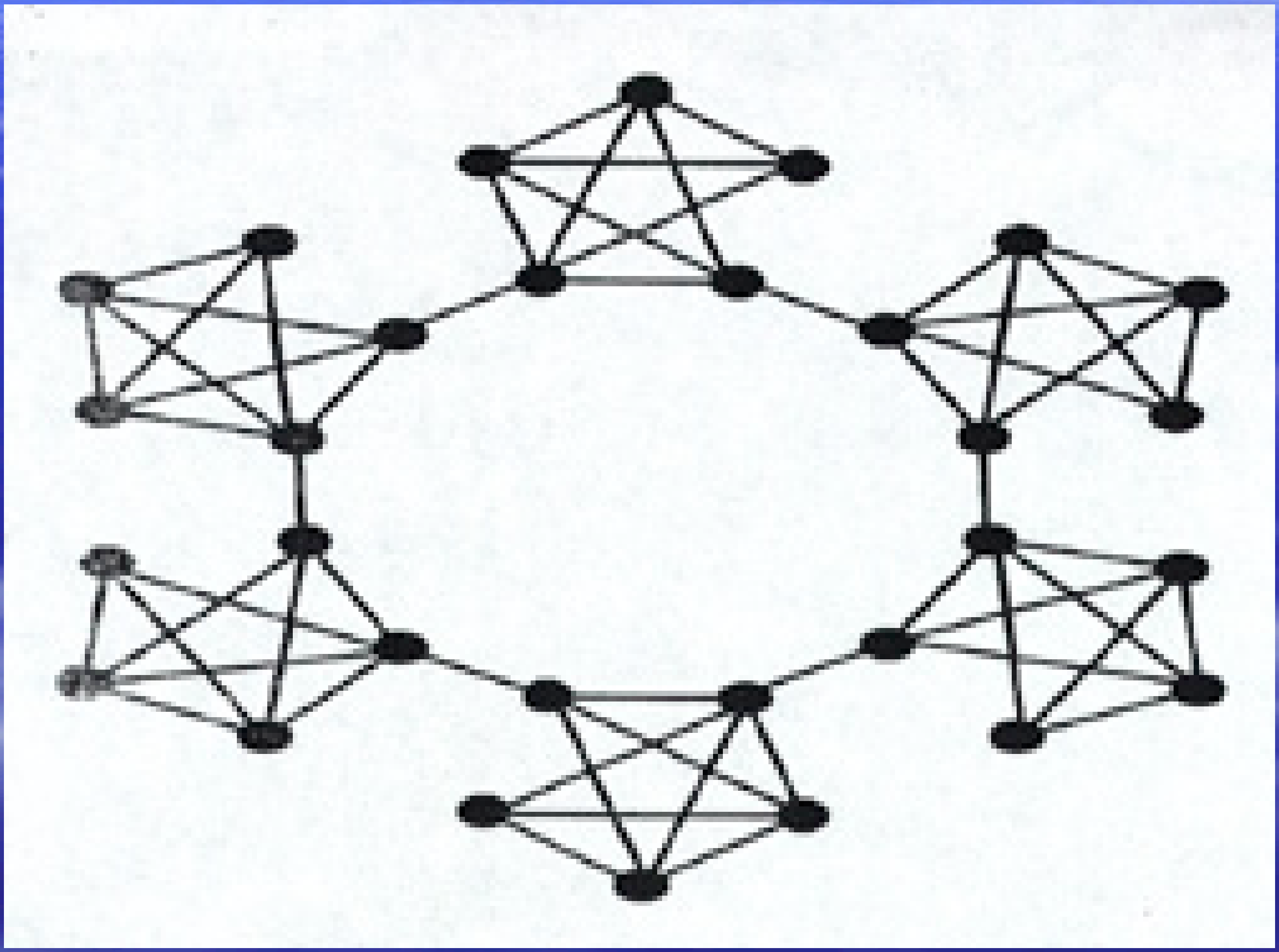
Where'd it come from?

- Stanley Milgram "Small World Study" 1967, published in Psychology Today, coined the phrase "six degrees of separation" applying it to the American population.
- Quickly was adopted into popular culture and applied to the whole world.

Duncan Watts arrives... Mathematical Motivation

- Ordered to Random spectrum of Networks(graphs).
- Ordered graphs and random graphs are generally both well understood and readily defined.
- Region between the two is not well understood and small-world graphs appear to be right in the middle.

Connected Cave Man



Toward a Mathematical Definition

- To be of interest a small world graph must be :

a) *highly clustered* (i.e. you have many close friends who are all friends of each other), and

b) *sparse* (it is a big world and not everyone knows everyone)

Yet there is only a small degree of separation between any two people.

-Model

$R^{i,j} =$ **(5)** where $R^{i,j}$ = a measure of vertex i's propensity to conn

$$R^{i,j} = \begin{cases} 1 & m_{i,j} \geq k \\ [\frac{m_{i,j}}{k}]^{\alpha}(1 - p) + p & k > m_{i,j} > 0 \\ p & m_{i,j} = 0 \end{cases}$$

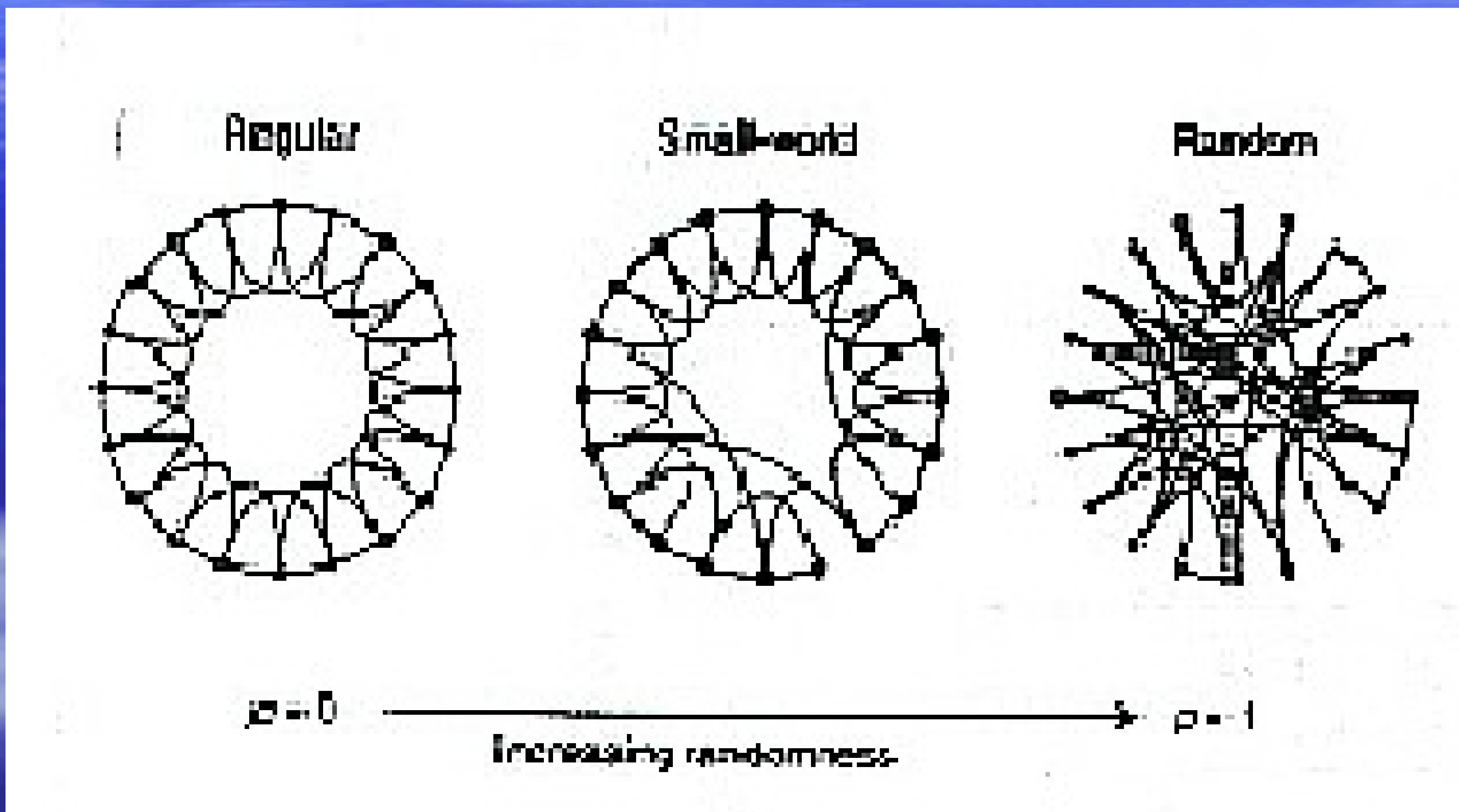
α

α .

Need for a substrate

- Without a substrate a caveman effect occurs for low □ and small disconnected graphs occur (infinite characteristic length).
- Watts makes a restriction on the model that graphs to be considered must be connected. He uses a ring substrate for the □-model.

□-Model



□ and □ models

- **Definition 3.1.4.** Given a graph of M = (k·n)/2 edges, the fraction of those edges that are shortcuts is denoted by □.
- **Definition 3.1.6.** □ is the fraction of all pairs of vertices that are not connected and have one and only one common neighbour.

Do they really exist?

- Kevin Bacon game (Rod Steiger)
- Power Grid
- the C. elegans (Caenorhabditis elegans)worm

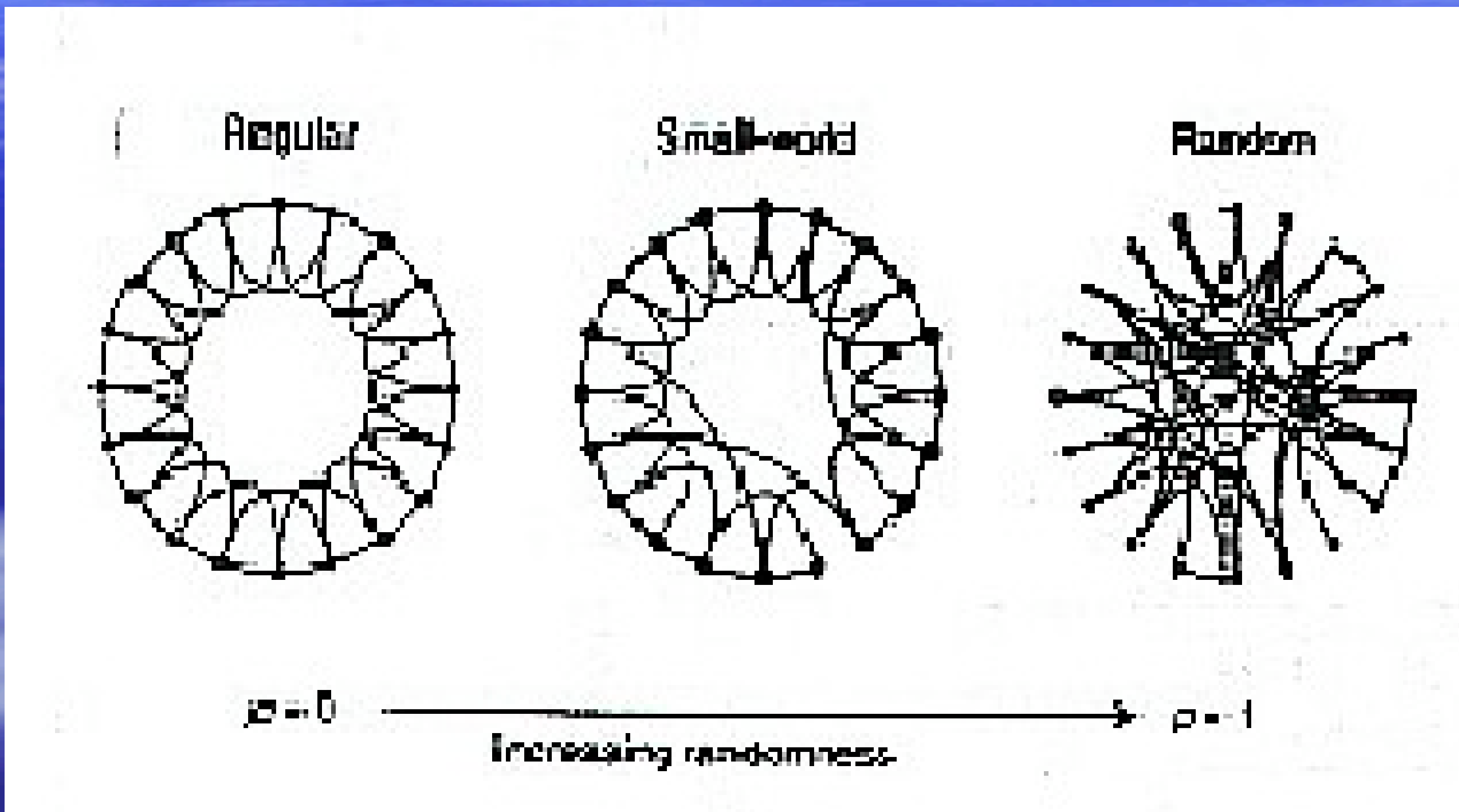
Dynamic Systems

- Disease spreading (susceptible, infectious, removed)
- Topology is important: Small world graphs seem to be in the critical area where the disease will begin to take over the whole population.

As a Computational Model

- Remember cellular automata
- They have an ordered lattice structure.

The -Model revisited



Density and Synchronization Problems

- Majority rules doesn't work for cellular automata, but does for small world graphs. Therein can treat each node as a simple CPU.
- For both problems small world graphs had better results (for density 0.9 versus 0.769)

- For other problems similar to cellular automata it is hard to see an approach to a solution.
- **Insight:** Should probably apply genetic algorithms not only to the rule set but also to all the possible connectivities.

Is it a small world after all?

- Milgram Experiment revisited.
- Send a package from some random person to some other random destination person in America.
- Only 30% made it through.

Questions?

Fin.