Cellular Automata

and beyond …

The World of Simple Programs

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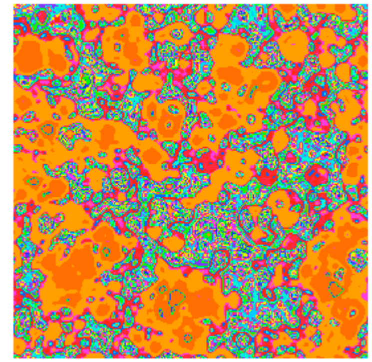
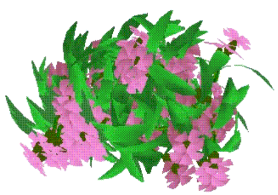
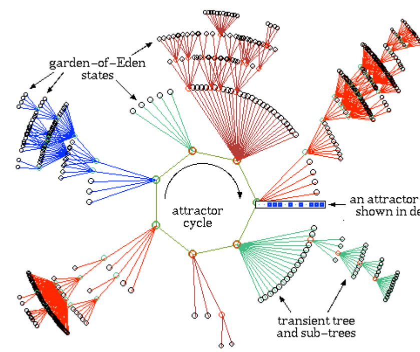
Cellular Automata



Lindenmayer
Systems



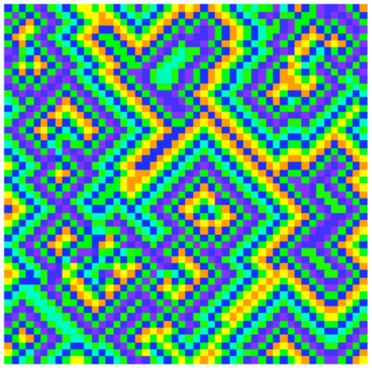
Random Boolean
Networks

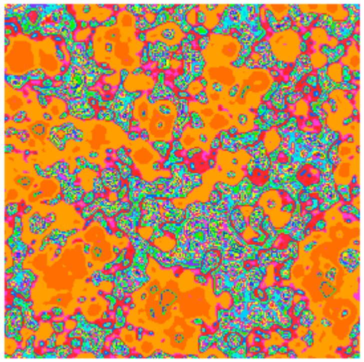


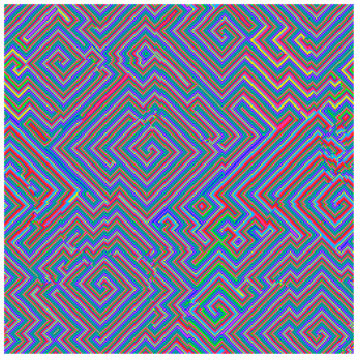
Classifier Systems

Cellular Automata

Global Effects from Local Rules







Cellular Automata

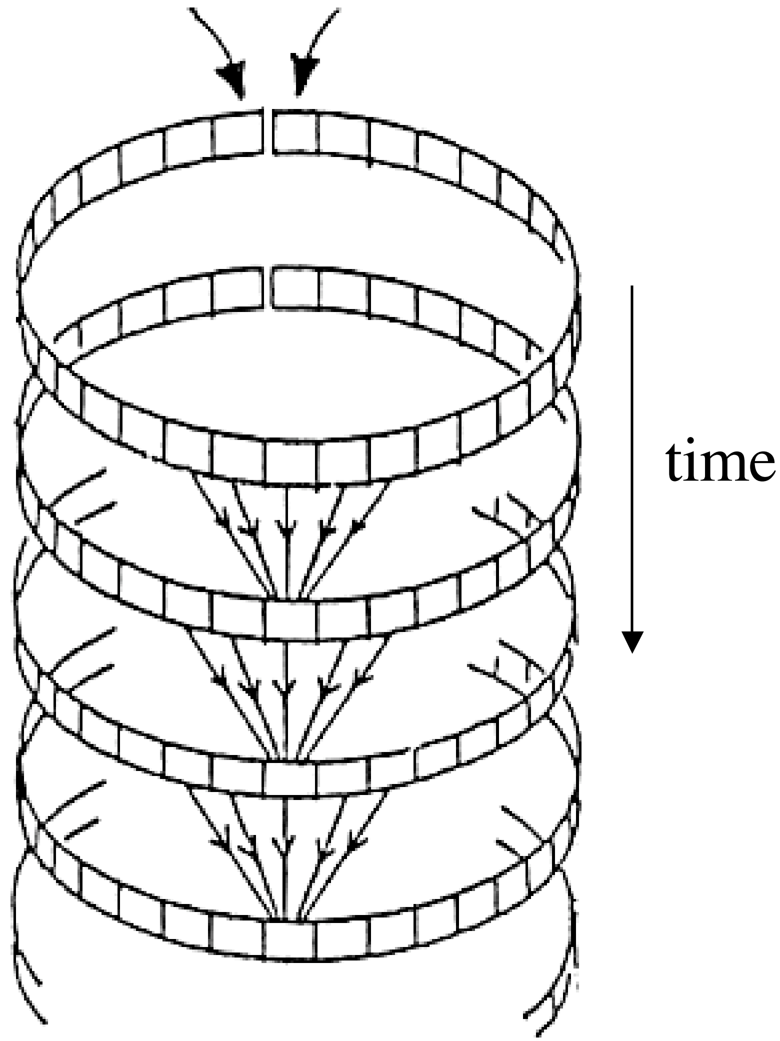
- The CA space is a lattice of cells (usually 1D, 2D, 3D) with a particular geometry.
- Each cell contains a variable from a limited range of values (e.g., 0 and 1).
- All cells update synchronously.
- All cells use the same updating rule (in uniform CA), depending only on local relations.
- Time advances in discrete steps.

One-dimensional Finite CA Architecture

- Neighbourhood size:
  K = 5

  local connections per cell

- Synchronous update in discrete time steps



A. Wuensche: The Ghost in the Machine, Artificial Life III, 1994.

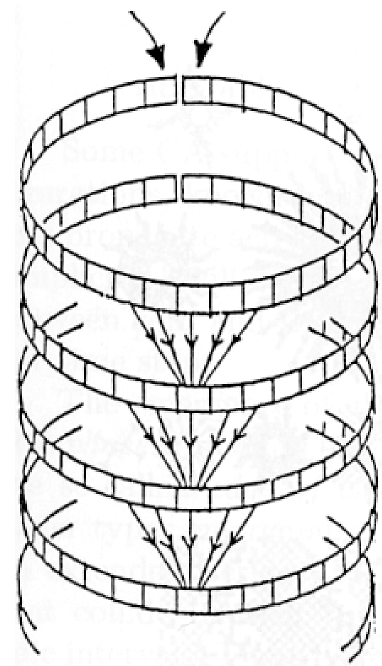
Time Evolution of Cell *i* with *K*-Neighbourhood

$$C_i^{(t+1)} = f(C_{i-[K/2]}^{(t)}, \ldots, C_{i-1}^{(t)}, C_i^{(t)}, C_{i+1}^{(t)}, \ldots, C_{i+[K/2]}^{(t)})$$

With periodic boundary conditions:

$$x < 1: C_x = C_{N+x}$$

$$x > N: C_x = C_{x-N}$$



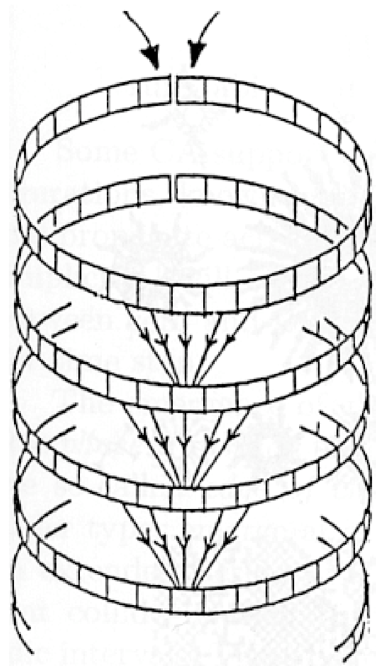
Value Range and Update Rules

- For V different states (= values) per cell there are V^K permuations of values in a neighbourhood of size K.

- The update function f can be implemented as a lookup table with V^K entries, giving V^{V^K} possible rules.

V^K {

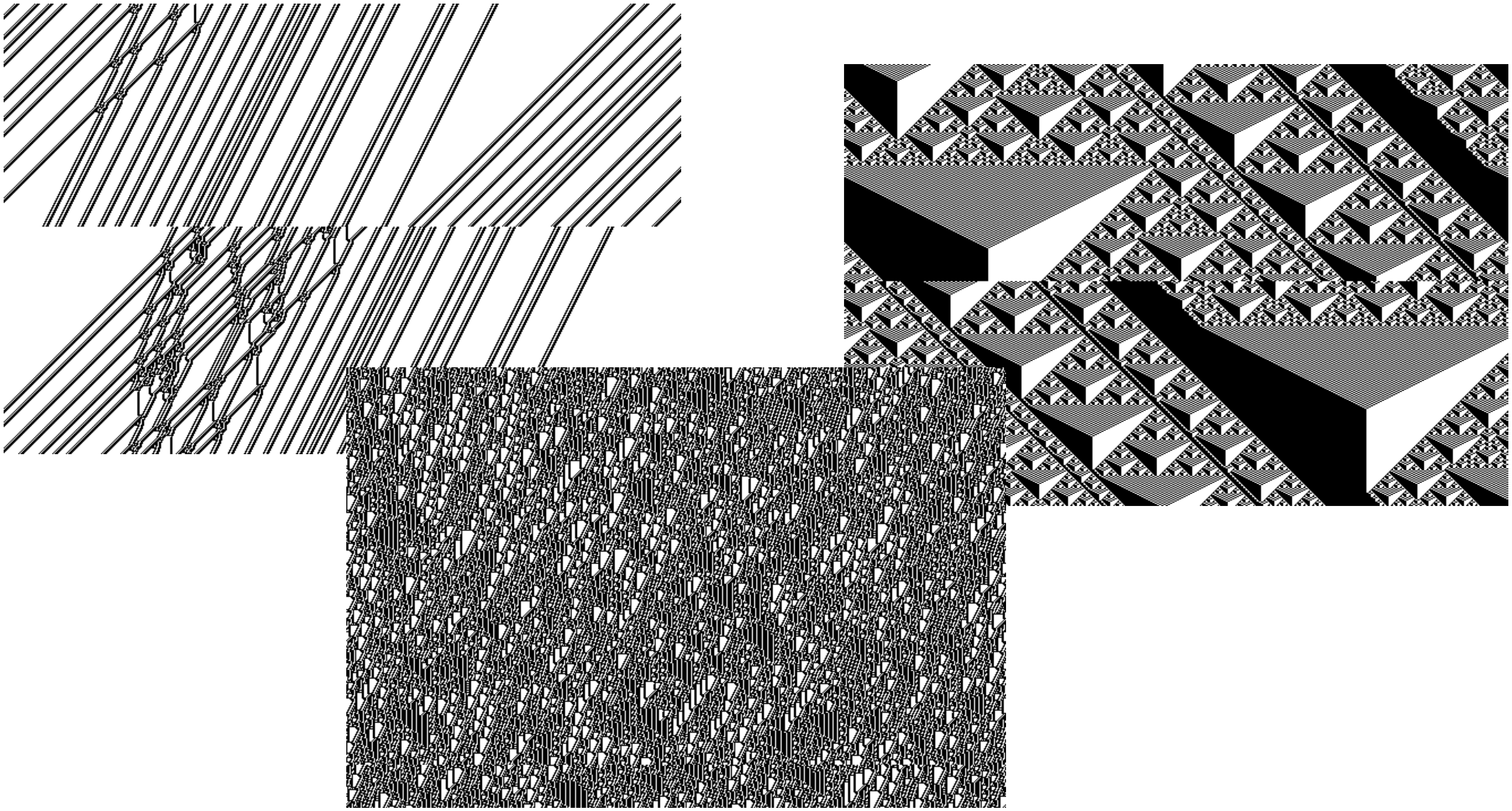
00000: 1 … V
00001: _
00010: _
…
11110: _
11111: _



v	K	v^K	$V^{v\wedge K}$
2	3	8	256
2	5	32	4.3×10^{9}
2	7	128	3.4×10^{38}
2	9	512	1.3×10^{154}

Cellular Automata: Local Rules — Global Effects

Demos



History of Cellular Automata

- Alternative names:
  - Tesselation automata
  - Cellular spaces
  - Iterative automata
  - Homogeneous structures
  - Universal spaces
- John von Neumann (1947)
  - Tries to develop abstract model of self-reproduction in biology (from investigations in cybernetics; Norbert Wiener)
- J. von Neumann & Stanislaw Ulam (1951)
  - 2D self-reproducing cellular automaton
  - 29 states per cell
  - Complicated rules
  - 200,000 cell configuration
  - (Details filled in by Arthur Burks in 1960s.)

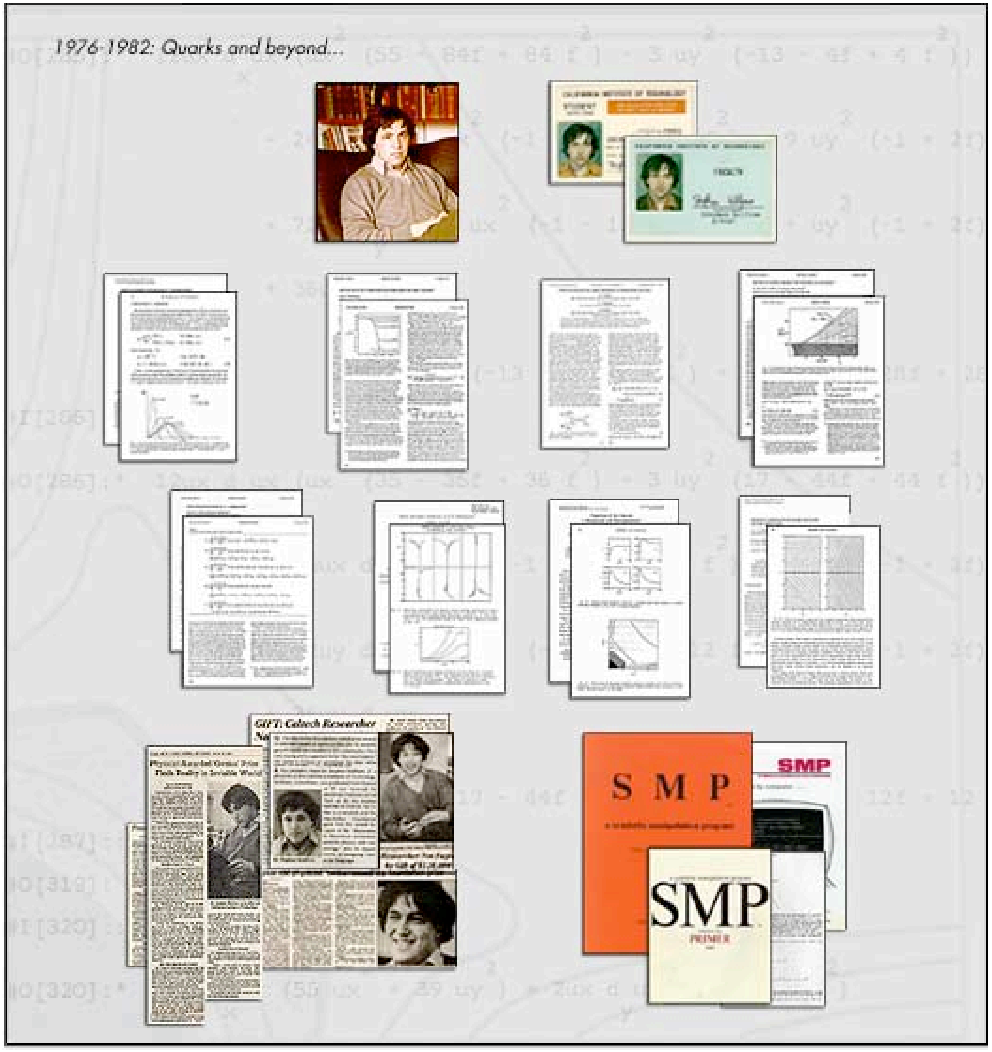
History of Cellular Automata (2)

- Threads emerging from J. von Neumann’s work:
  - Self-reproducing automata (spacecraft!)
  - Mathematical studies of the essence of
    - Self-reproduction and
    - Universal computation.
- CAs as Parallel Computers (end of 1950s / 1960s)
  - Theorems about CAs (analogies to Turing machines) and their formal computational capabilities
  - Connecting CAs to mathematical discussions of dynamical systems (e.g., fluid dynamics, gases, multi-particle systems)
- 1D and 2D CAs used in electronic devices (1950s)
  - Digital image processing (with so-called cellular logic systems)
  - Optical character recognition
  - Microscopic particle counting
  - Noise removal

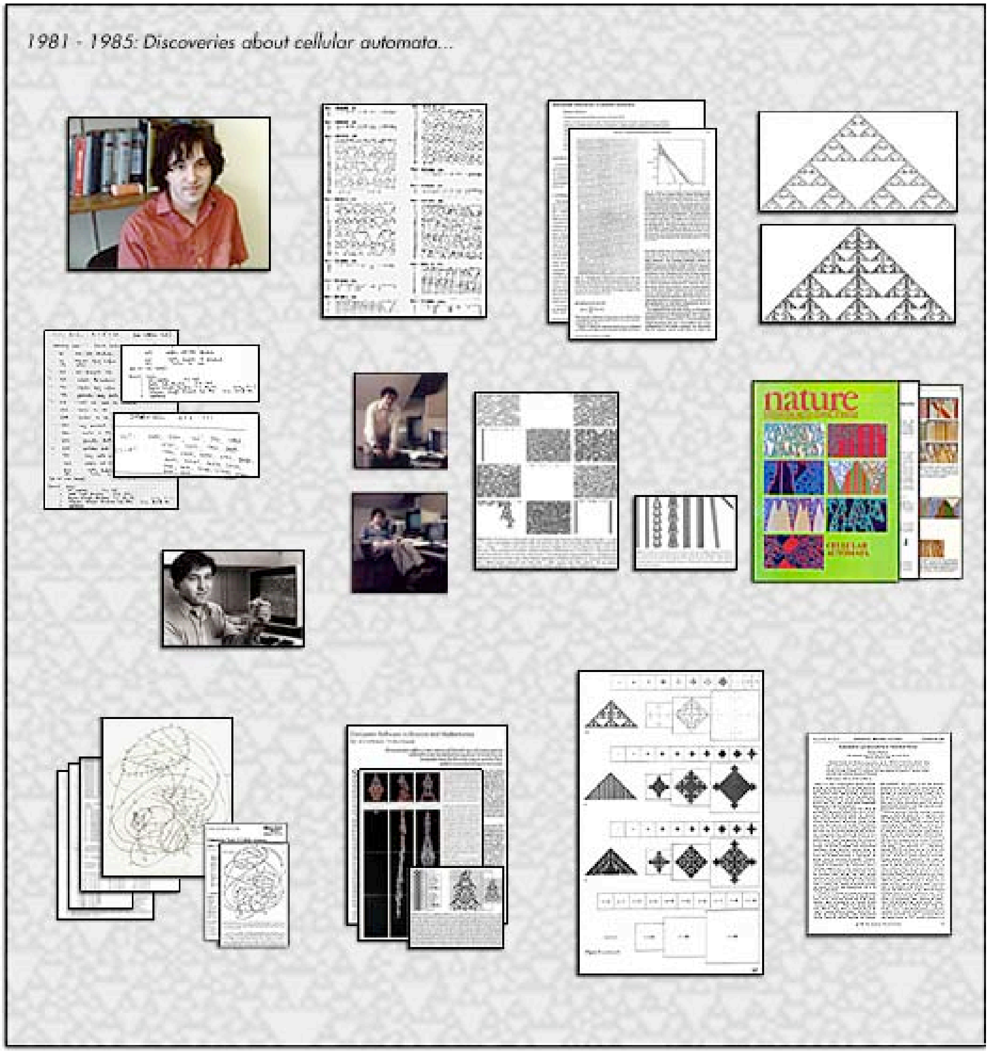
History of Cellular Automata (3)

- Stansilaw Ulam at Los Alamos Laboratories
  - 2D cellular automata to produce recursively defined geometrical objects (evolution from a single black cell)
  - Explorations of simple growth rules
- Specific types of Cas (1950s/60s)
  - 1D: optimization of circuits for arithmetic and other operations
  - 2D:
    - Neural networks with neuron cells arranged on a grid
    - Active media: reaction-diffusion processes
- John Horton Conway (1970s)
  - Game of Life (on a 2D grid)
  - Popularized by Martin Gardner: *Scientific American*

Stephen Wolfram's World of CAs

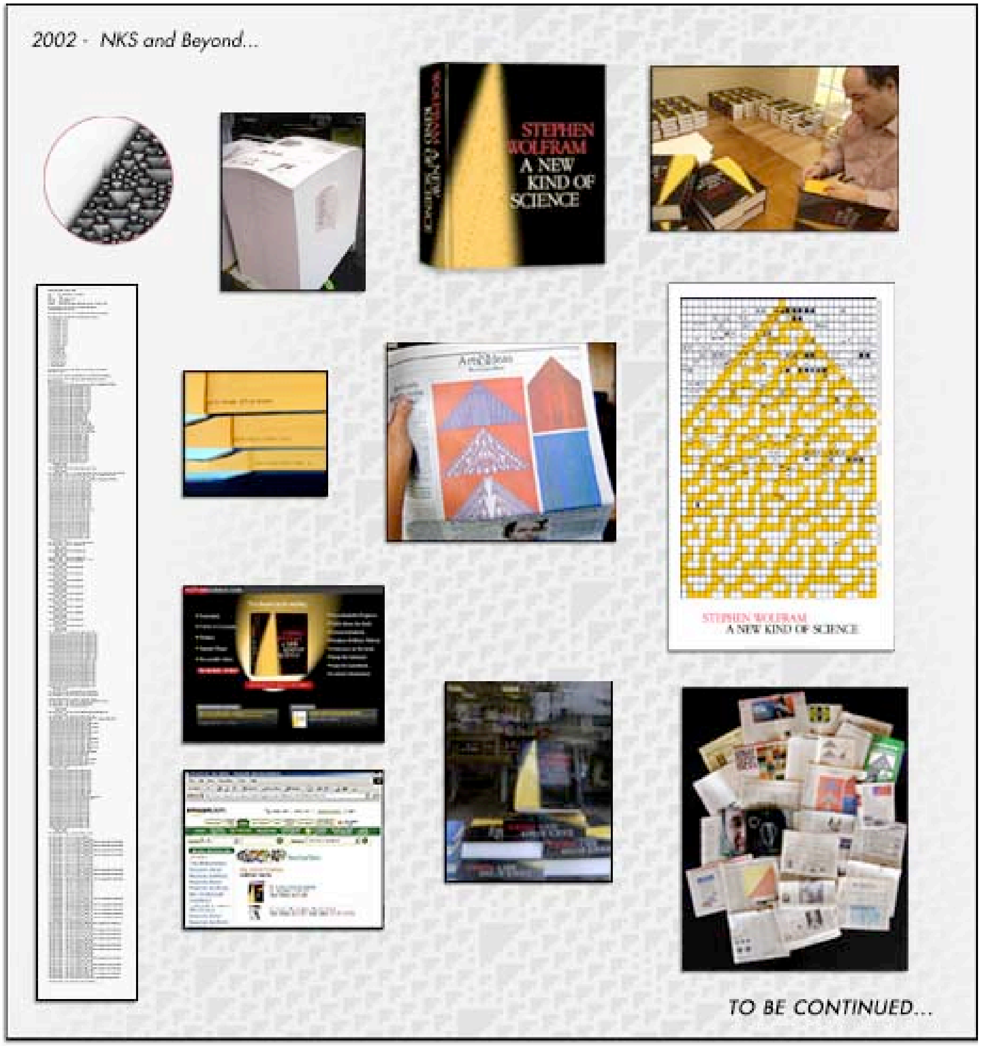


Stephen Wolfram's World of CAs



Stephen Wolfram's World of CAs

Stephen Wolfram's World of CAs



Example Update Rule

- V = 2, K = 3
- The rule table for rule 30:

111	110	101	100	011	010	001	000
↓	↓	↓	↓	↓	↓	↓	↓
0	0	0	1	1	1	1	0
128	64	32	16	8	4	2	1

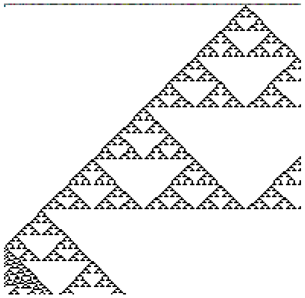
16 + 8 + 4 + 2 = 30

See examples ...

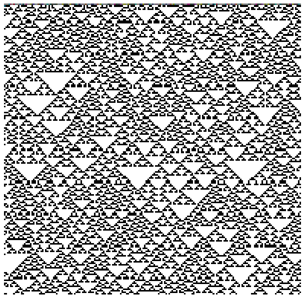
CA Demos

- *Evolvica* CA Notebooks

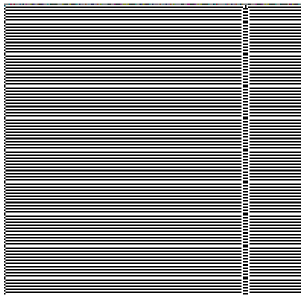
22: {0, 0, 0, 1, 0, 1, 1, 0}



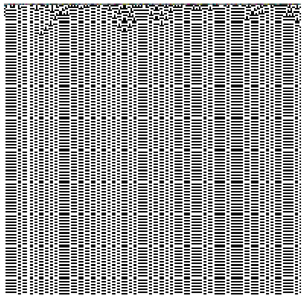
22: {0, 0, 0, 1, 0, 1, 1, 0}



37: {0, 0, 1, 0, 0, 1, 0, 1}

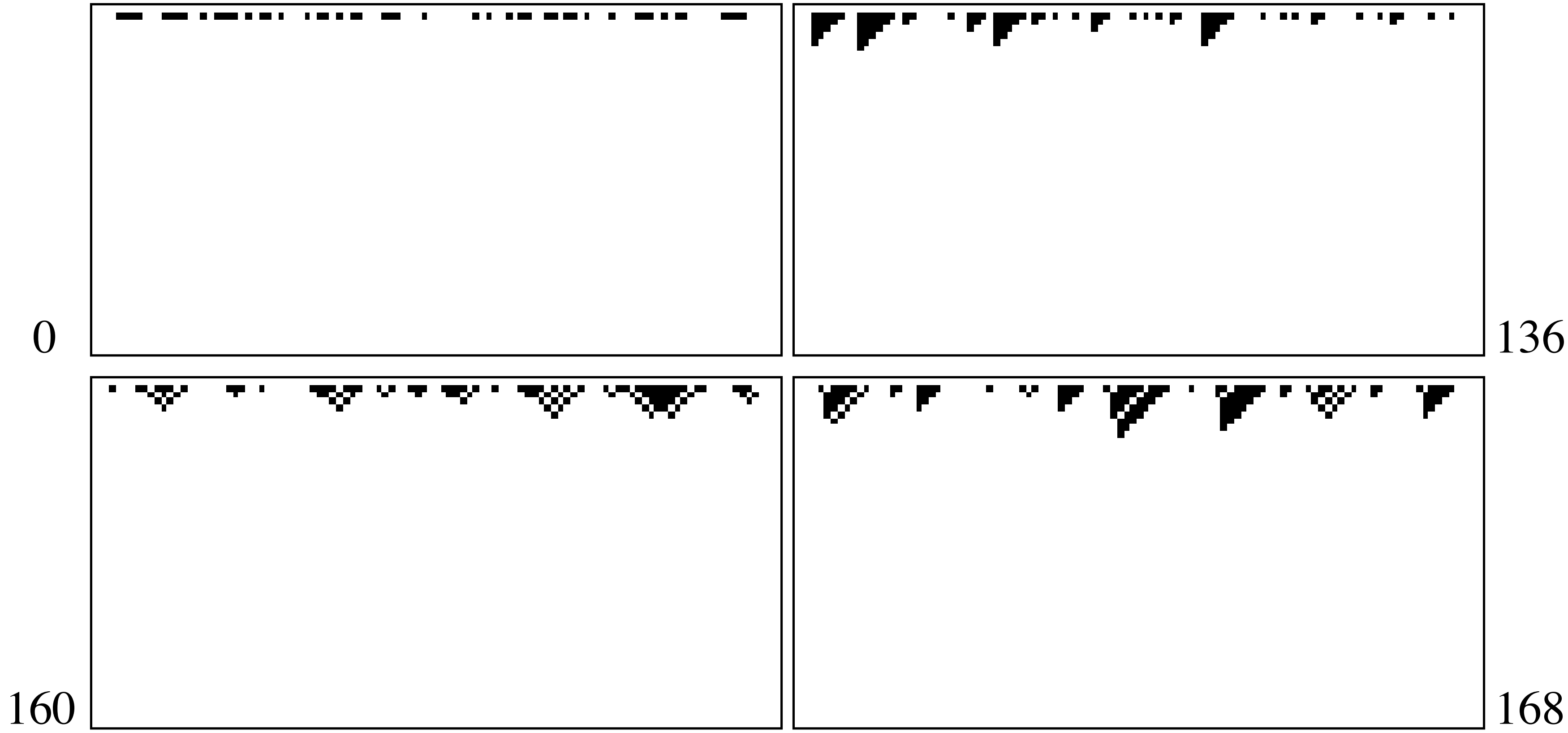


37: {0, 0, 1, 0, 0, 1, 0, 1}



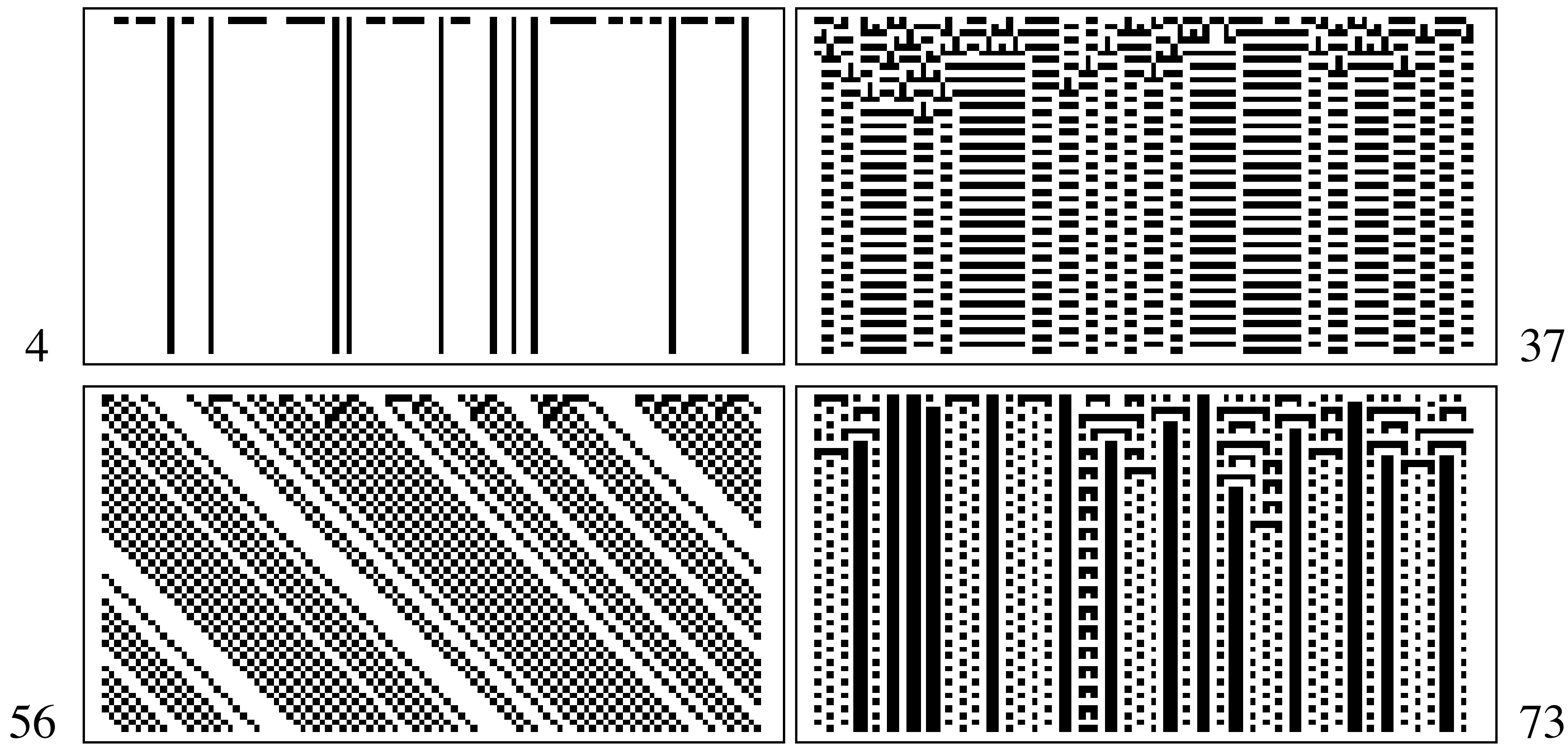
Four Wolfram Classes of CA

- **Class 1**:
  A fixed, homogeneous, state is eventually reached
  (e.g., rules 0, 8, 128, 136, 160, 168).



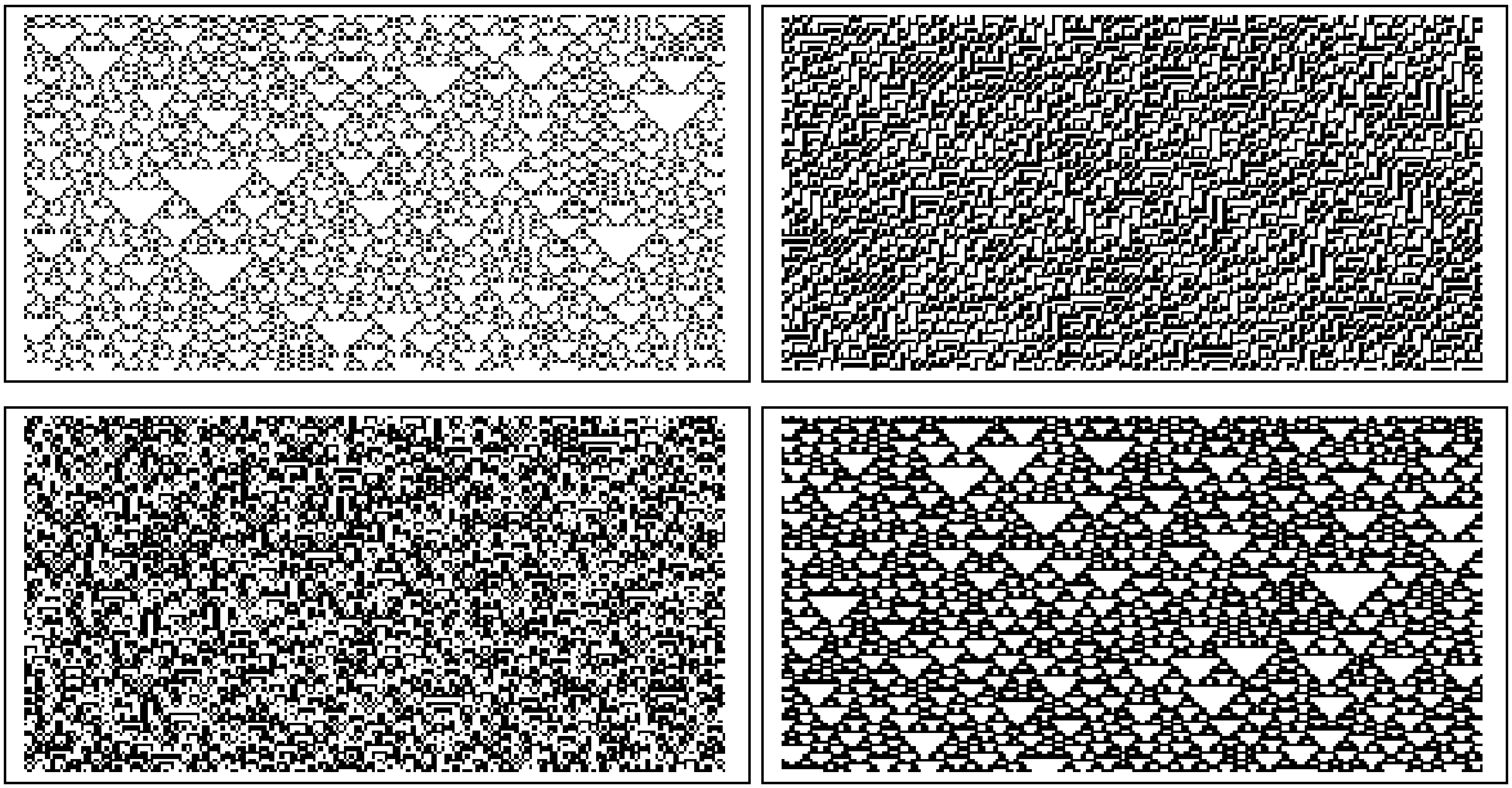
Four Wolfram Classes of CA

- **Class 2**:
  A pattern consisting of separated periodic regions is produced (e.g., rules 4, 37, 56, 73).



Four Wolfram Classes of CA

- **Class 3**:
  A chaotic, aperiodic, pattern is produced
  (e.g., rules 18, 45, 105, 126).

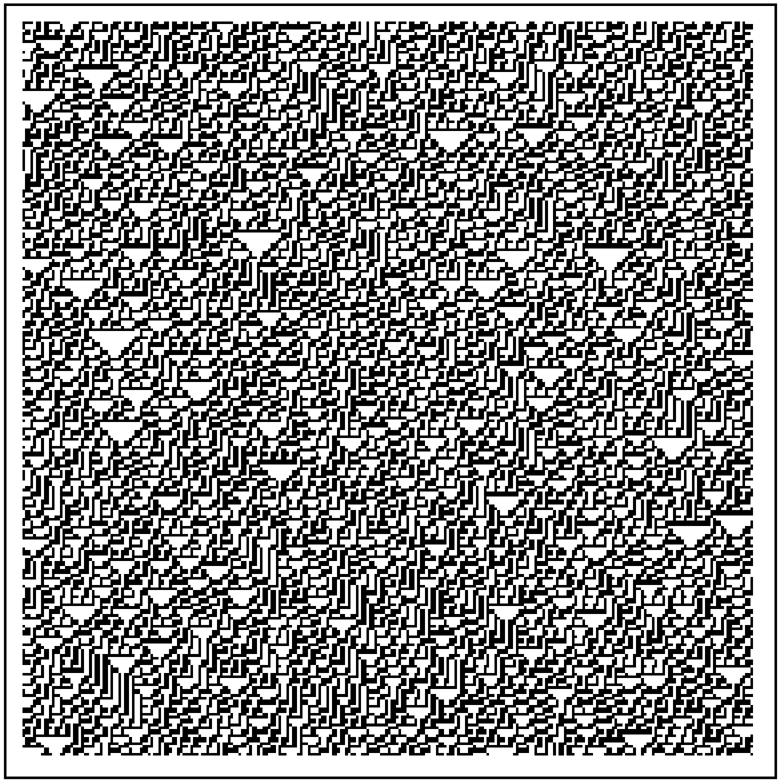
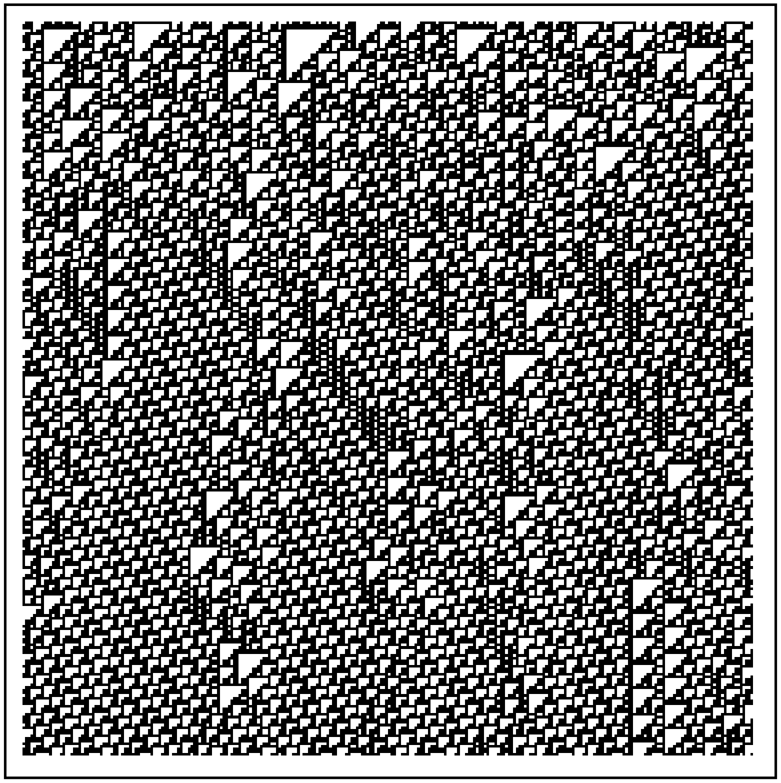


18 45

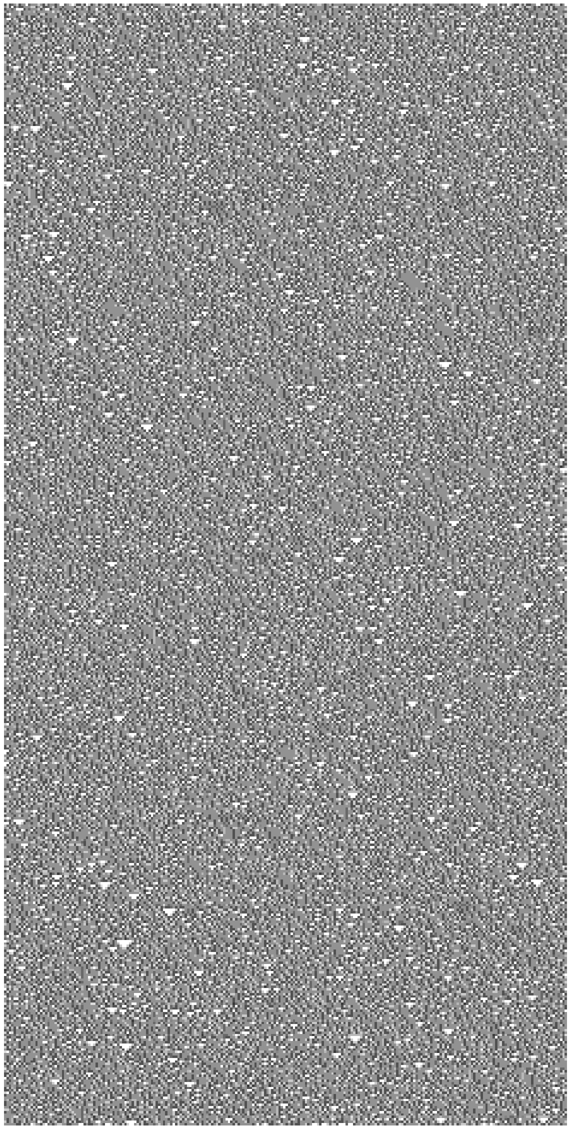
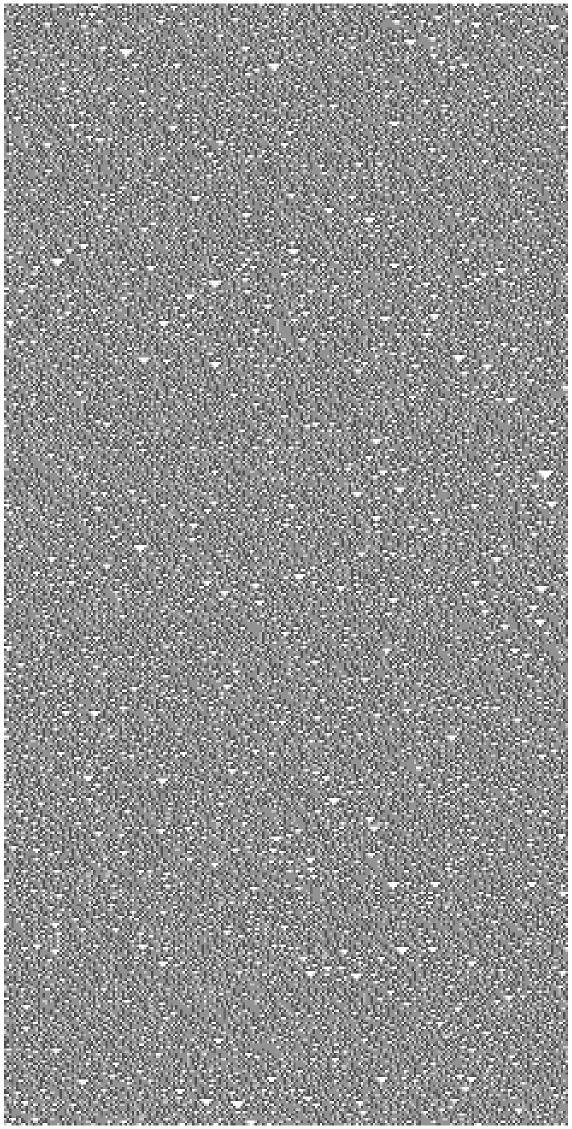
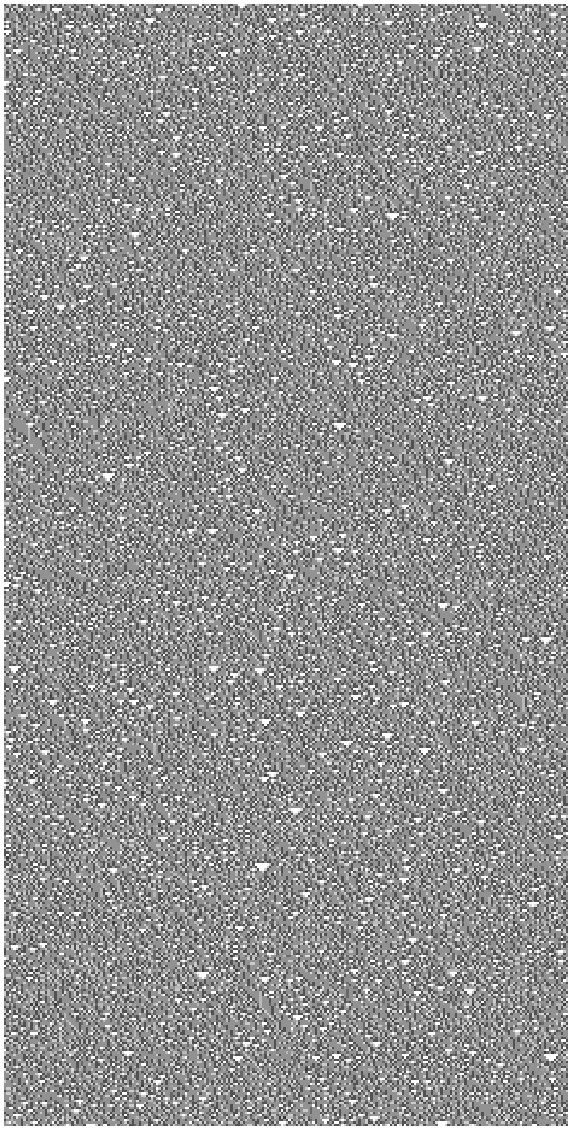
105 126

Four Wolfram Classes of CA

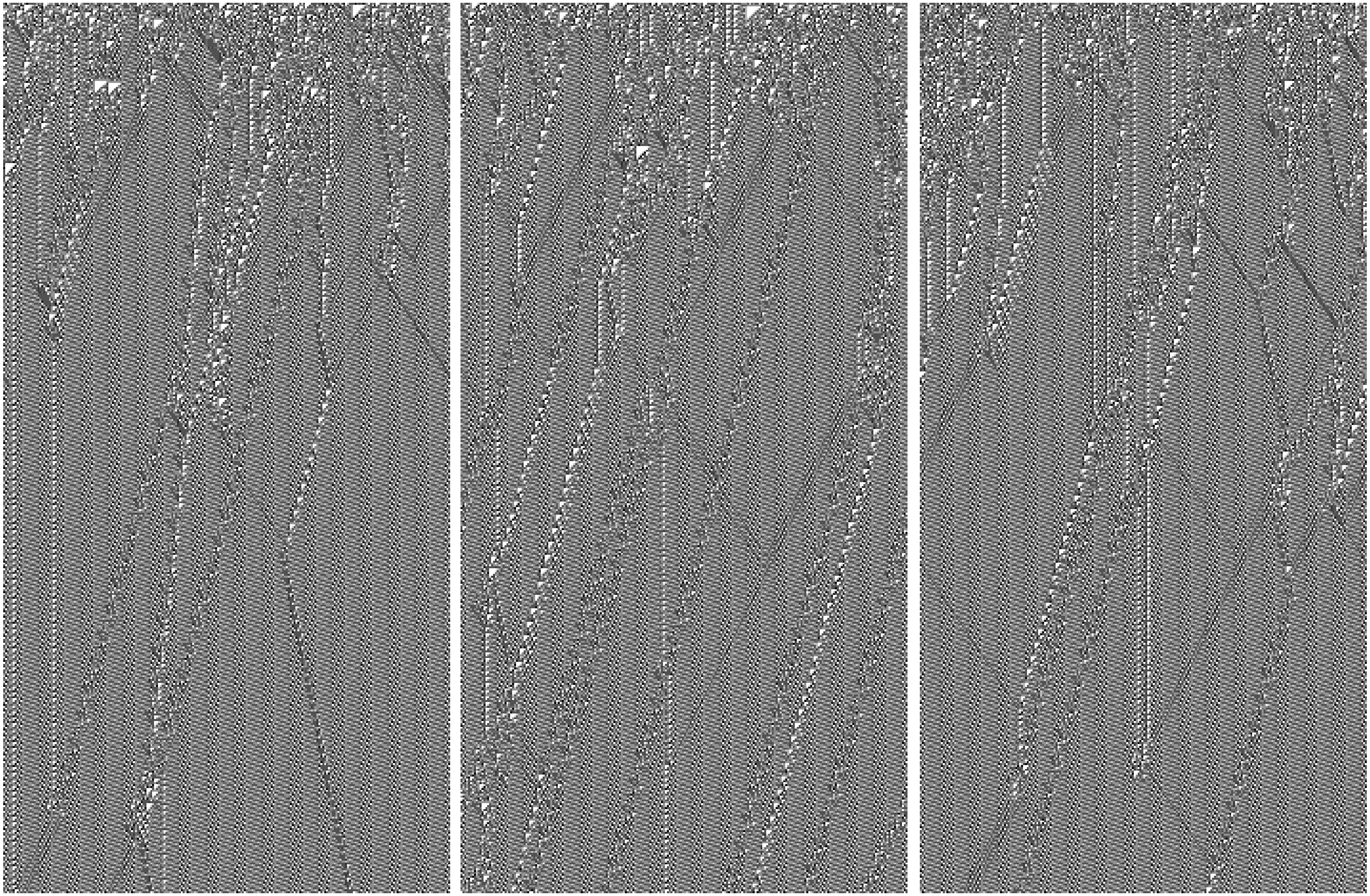
- **Class 4**:
  Complex, localized structures are generated (e.g., rules 30, 110).

30   110

Class 4: Rule 30

Class 4: Rule 110



Further Classifications of CA Evolution

- Wolfram classifies CAs according to the patterns they evolve:
  - 1. Pattern disappears with time.
  - 2. Pattern evolves to a fixed finite size.
  - 3. Pattern grows indefinitely at a fixed speed.
  - 4. Pattern grows and contracts irregularly.

–3/text.html: Fig. 1

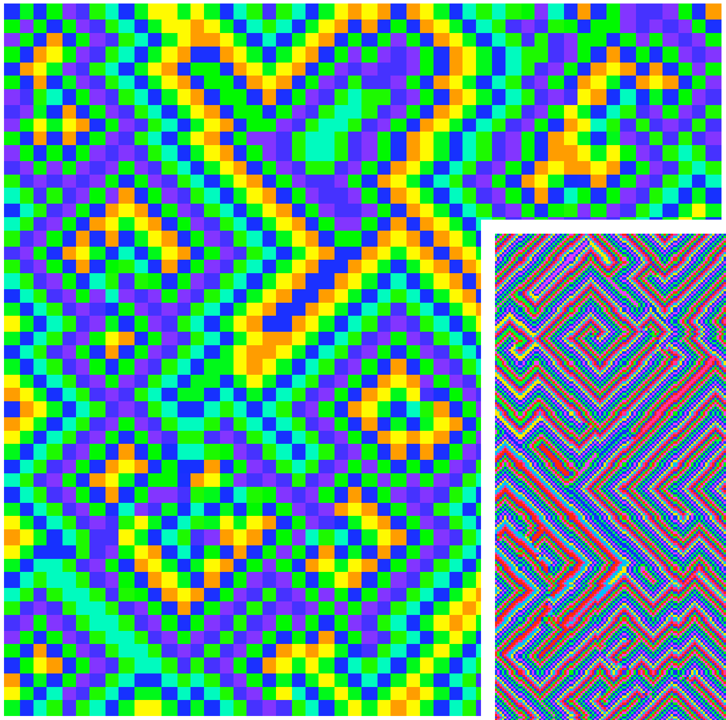
- Qualitative Classes
  - 1. Spatially homogeneous state
  - 2. Sequence of simple stable or periodic structures
  - 3. Chaotic aperiodic behaviour
  - 4. Complicated localized structures, some propagating

–85-cellular/7/text.html: Fig. 3 (first row)

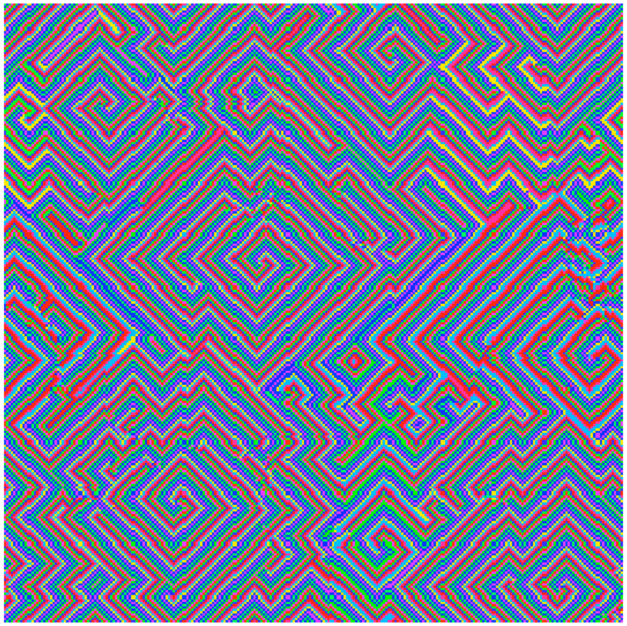
Further Classifications of CA Evolution (2)

- Classes from an Information Propagation Perspective
  - 1. No change in final state
  - 2. Changes only in a finite region
  - 3. Changes over an ever-increasing region
  - 4. Irregular changes
- Degrees of Predictability for the Outcome of the CA Evolution
  - 1. Entirely predictable, independent of initial state
  - 2. Local behavior predictable from local initial state
  - 3. Behavior depends on an ever-increasing initial region
  - 4. Behavior effectively unpredictable

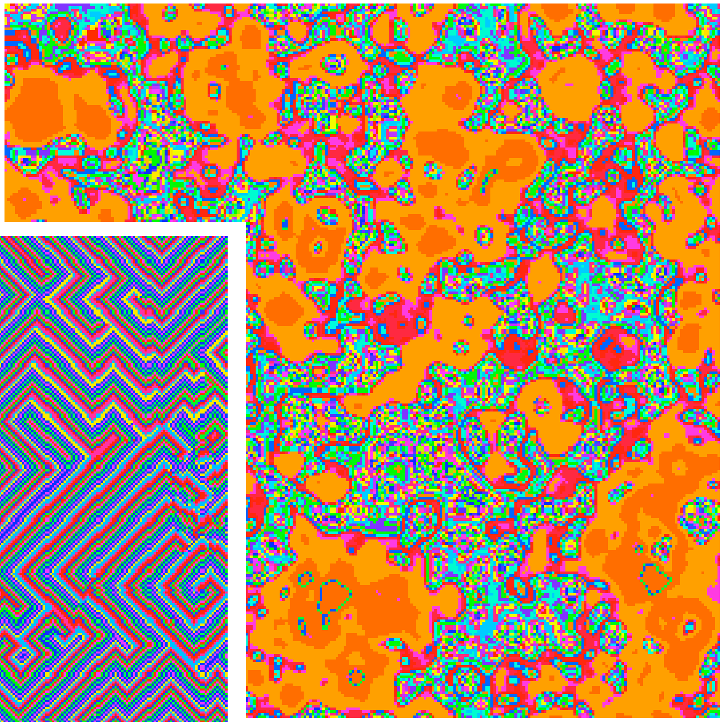
2-D CA: Emergent Pattern Formation in Excitable Media



Neuron excitation



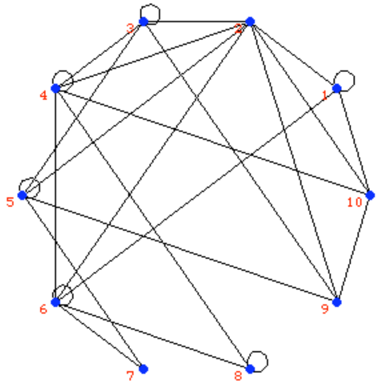
Neuron excitation (relaxed)

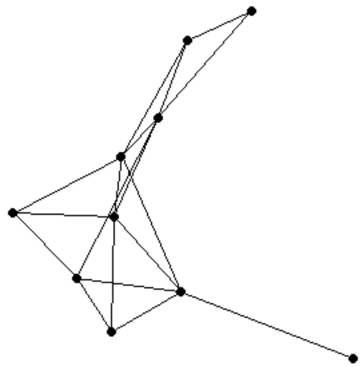


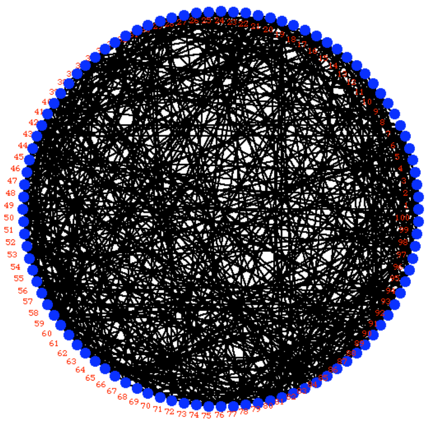
Hodgepodge

Random Boolean Networks

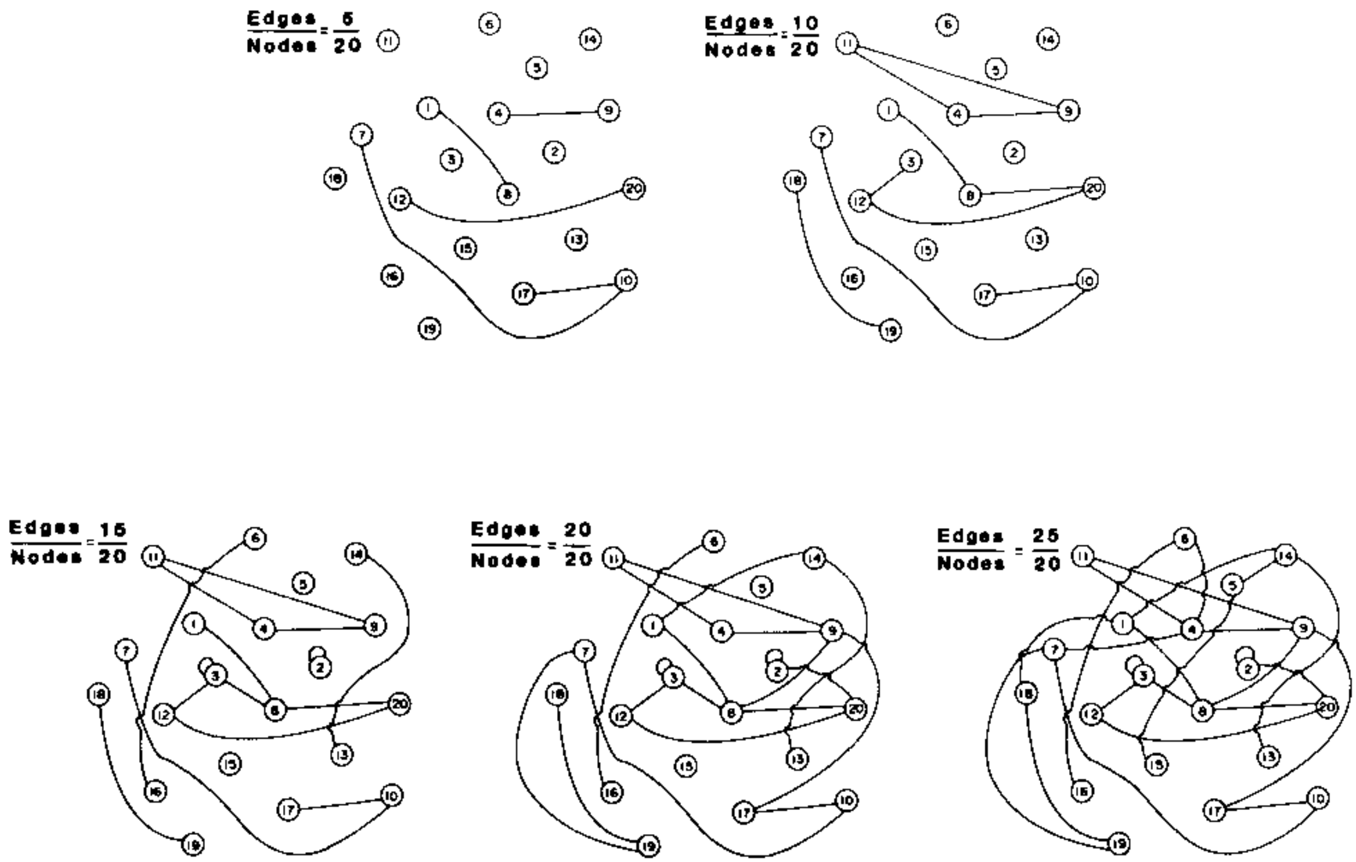
Generalized Cellular Automata







Crystallization of Connected Webs

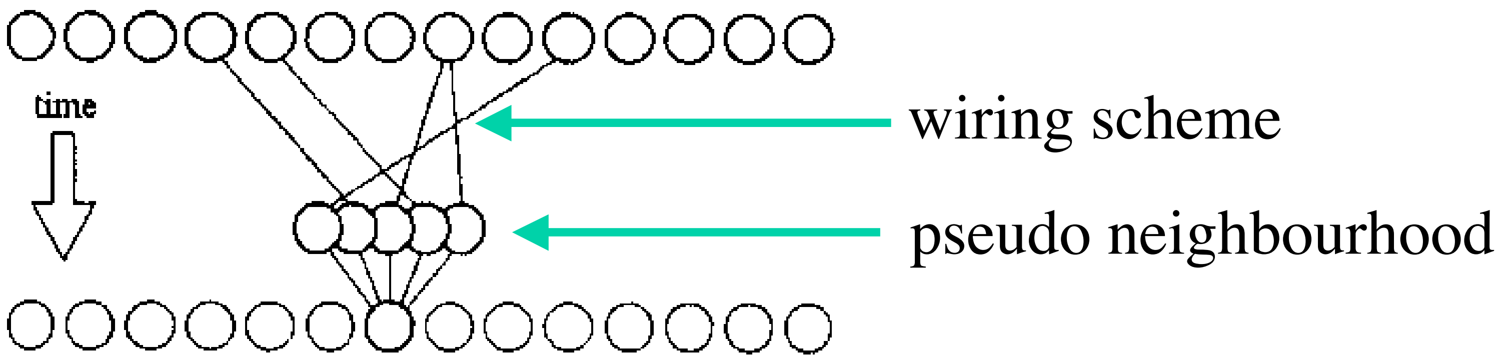


[S. Kauffman: At Home in the Universe]

Random Nets Demo

Random Network Architecture

Network at time *t*

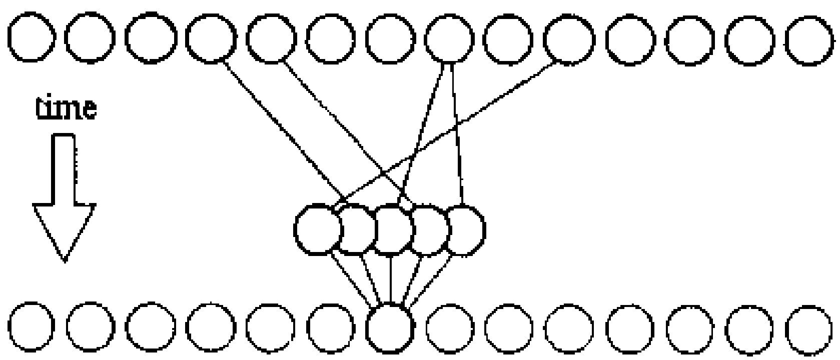


Network at time *t*+1

Time Evolution of the *i*-th Cell

- Cell i is connected to K cells $w_{i1}, w_{i2}, \ldots, w_{iK}$; with w_{ij} from $\{1,\ldots, N\}$.
- N^K possible alternative wiring options.
- Update rule for cell i:

$$C_i^{(t+1)} = f_i(C_{w_{i1}}^{(t)}, C_{w_{i2}}^{(t)}, \ldots, C_{w_{iK}}^{(t)})$$



Wiring/Rule Schemes

- A random network of size N with neighbourhood size K can be assigned

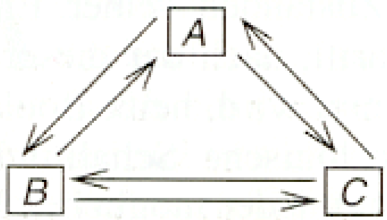
$$S = (N^K)^N \cdot (V^{V^K})^N$$

alternative wiring and rule schemes.

- Example:
  V = 2, N = 16, K = 15; S = 2^{832} .

States and Cycles

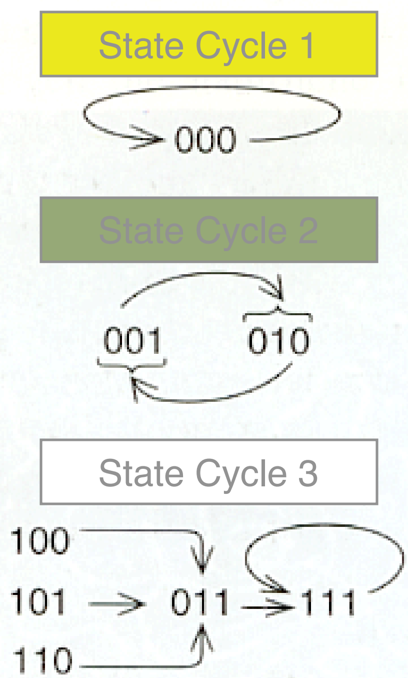
a



b

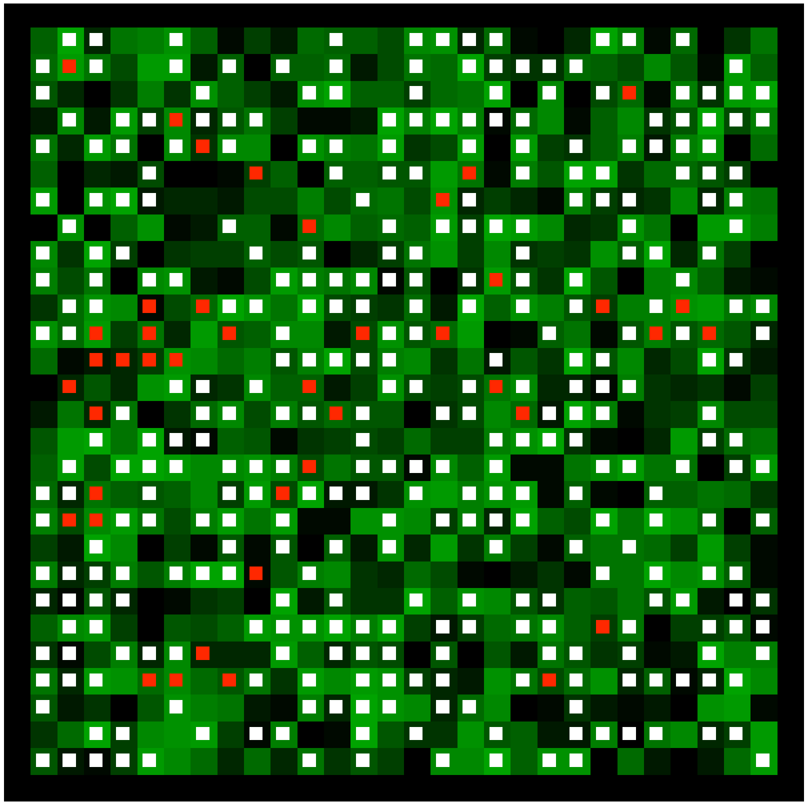
System State			Following State		
A	B	C	A	B	C
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	0	1	1
0	1	1	1	1	1
1	1	1	1	1	1

c

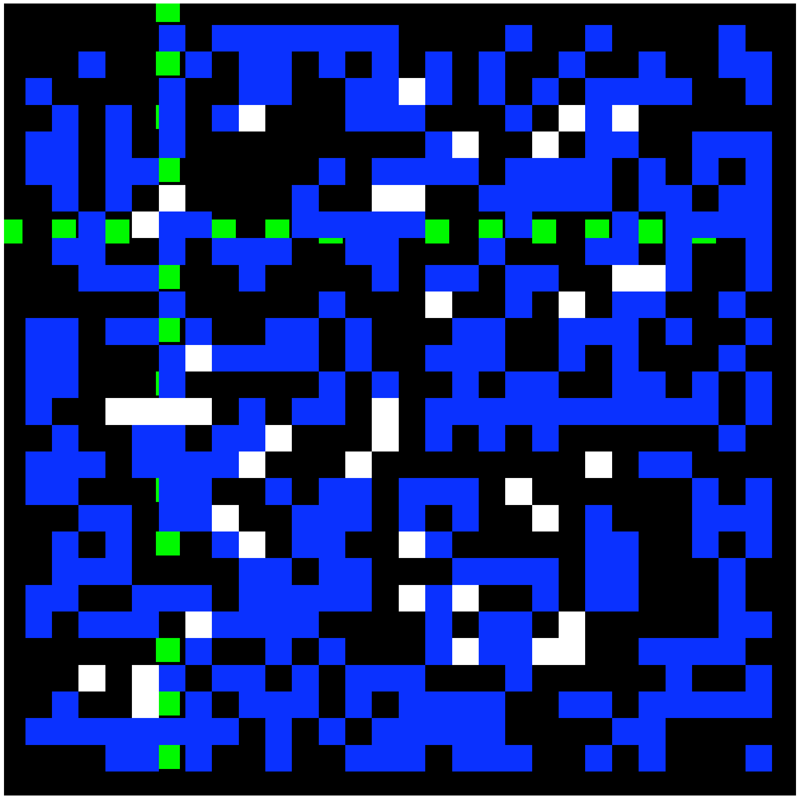


[S. Kauffman: Leben am Rande des Chaos]

Kauffman's Random Boolean Networks



Boolean functions represented by shades of green.
Stuart Kauffman used this network to investigate the interaction of proteins within living systems.

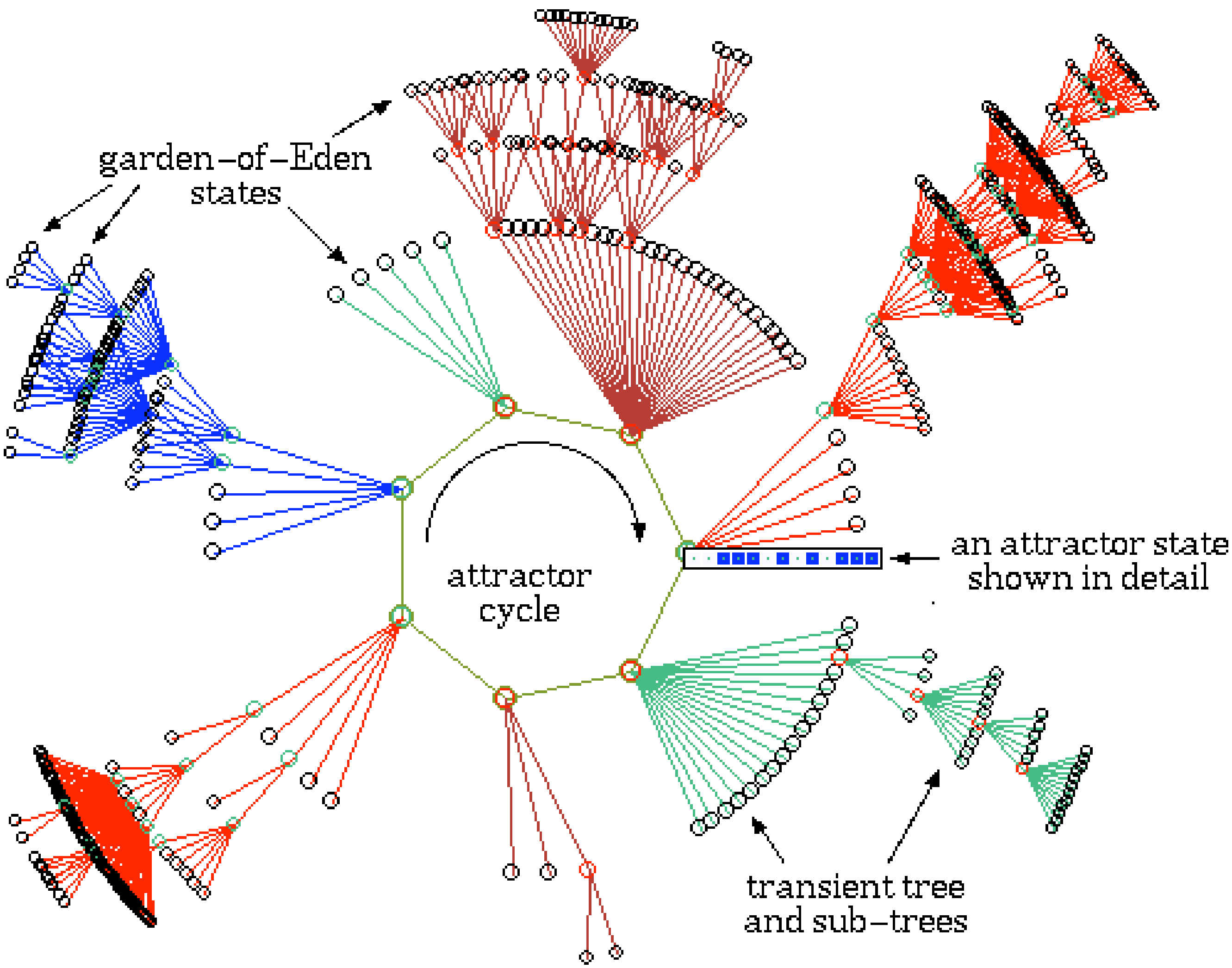


Binary values that have changed are white.
Unchanged values are blue.

These networks settle very quickly into an oscillatory state.

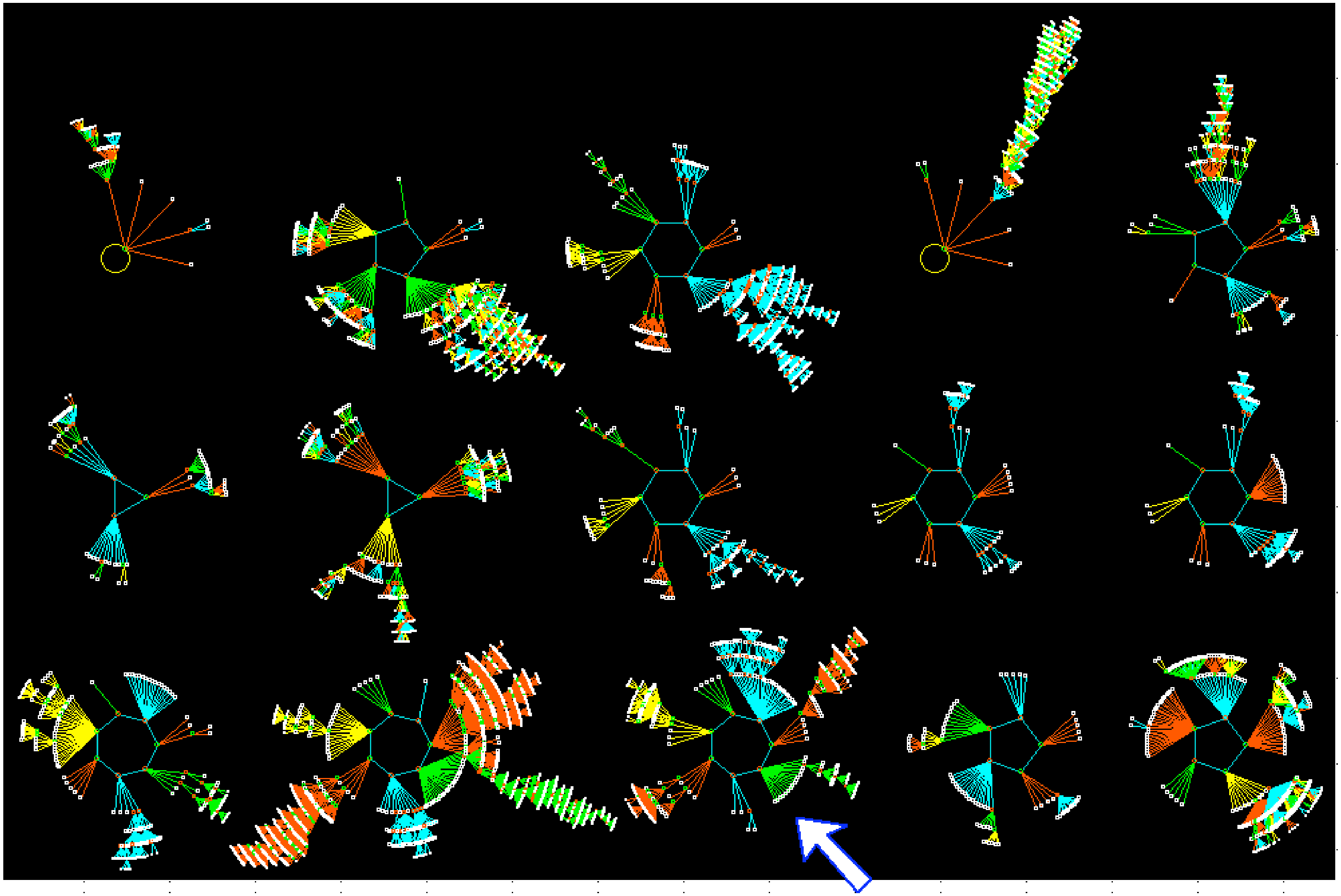
http://members.rogers.com/fmobrien/experiments/boolean_net/BooleanNetworkApplet_both.html

Attractor Cycles



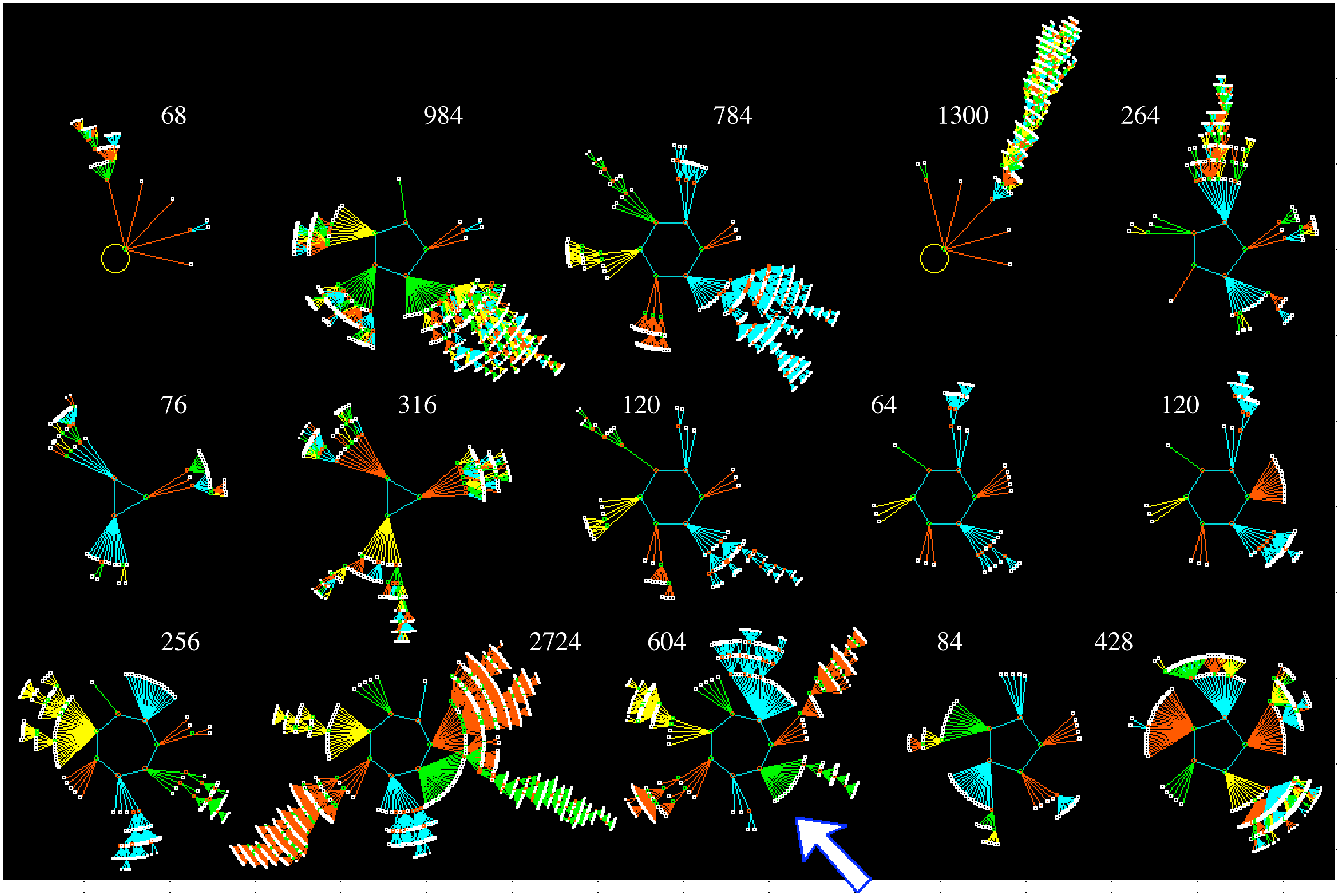
[A. Wuensche, Discrete Dynamics Lab]

Basin of Attraction Field



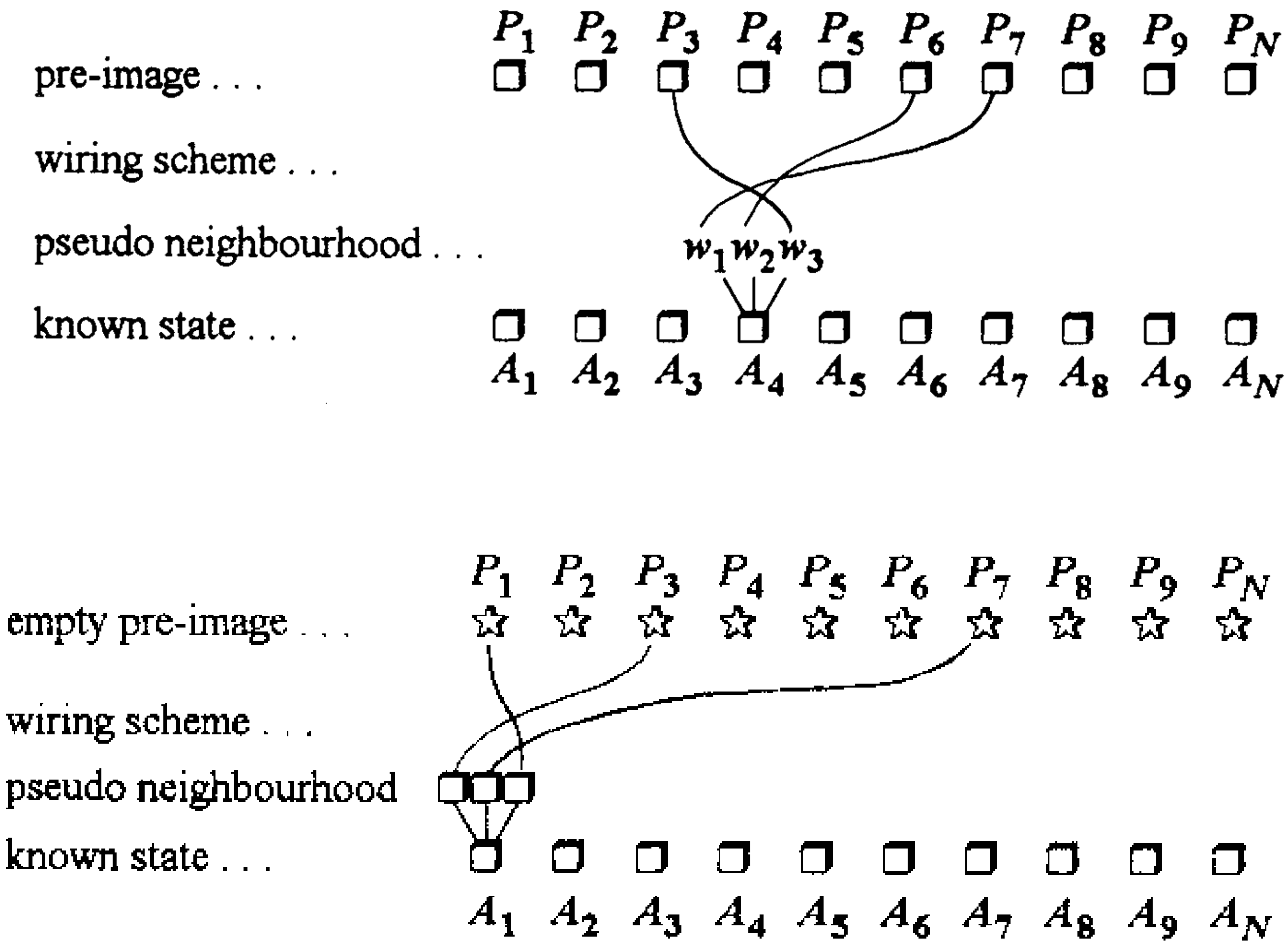
Nodes: n =13; Connectivity: k = 3; States: $2^{13} = 8192$ [A. Wuensche, Discrete Dynamics Lab]

Basin of Attraction Field



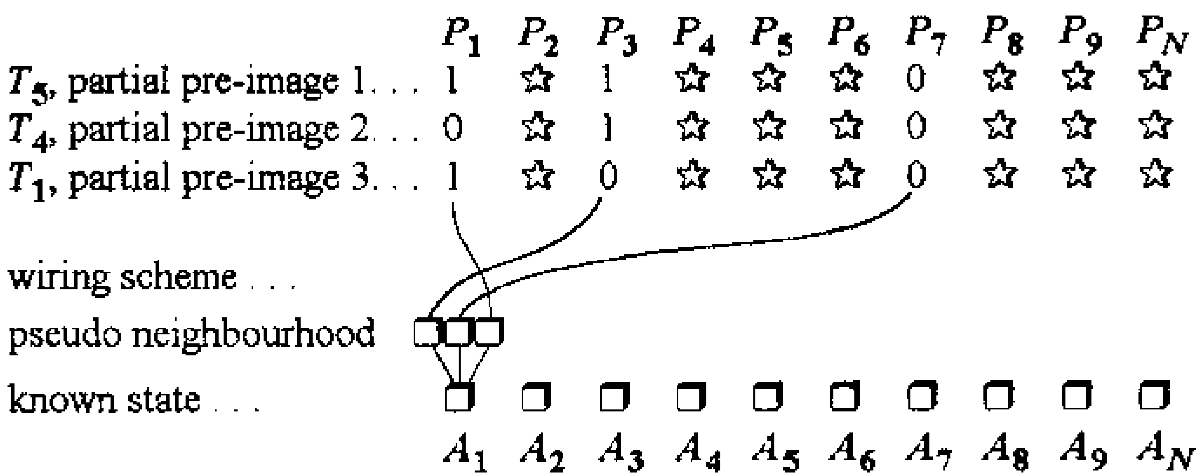
Nodes: n =13; Connectivity: k = 3; States: $2^{13} = 8192$ [A. Wuensche, Discrete Dynamics Lab]

Calculating Pre-Images

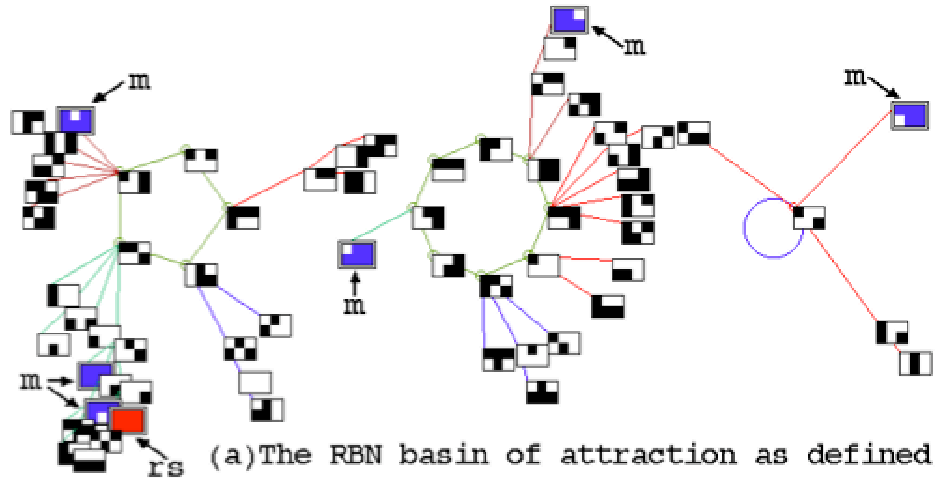


Calculating Pre-Images (2)

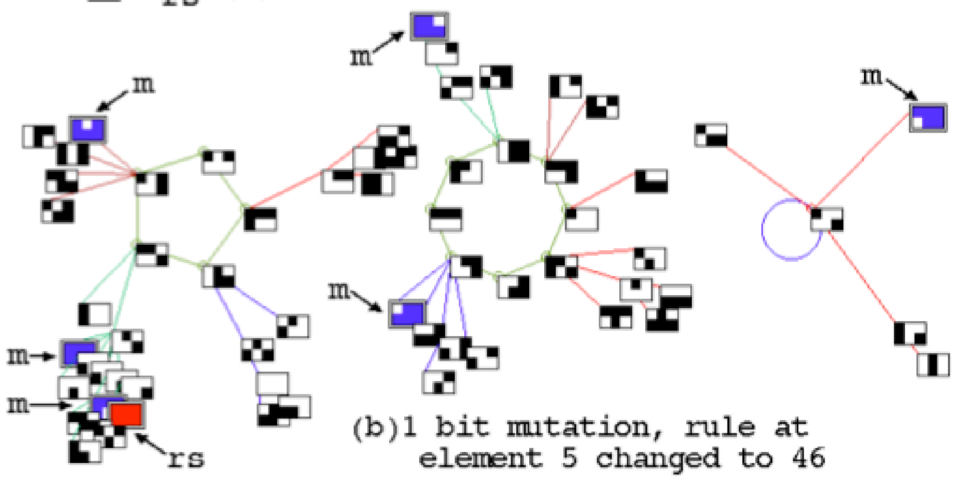
	111	110	101	100	011	010	001	000	...neighborhoods
rule table...	0	0	1	1	0	0	1	0	...outputs (0 or 1)
	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	



Mutations on Random Boolean Ntworks



(a)The RBN basin of attraction as defined



(b)1 bit mutation, rule at element 5 changed to 46

el.	wiring	rule
5	2,4,5	62
4	5,0,1	61
3	4,3,5	108
2	2,5,0	5
1	4,2,1	64
0	3,1,2	231

Figure 21: The basin of attraction field of (a) The RBN (n=6, k=3) as defined in the table (above), and (b) the RBN following a 1 bit mutation to one of its rules. Some differences in the fields are evident. The result or a 1 bit perturbation to a reference state of all 1s (rs) is indicated by its 1 bit mutants (m).

[A. Wuensche 98]

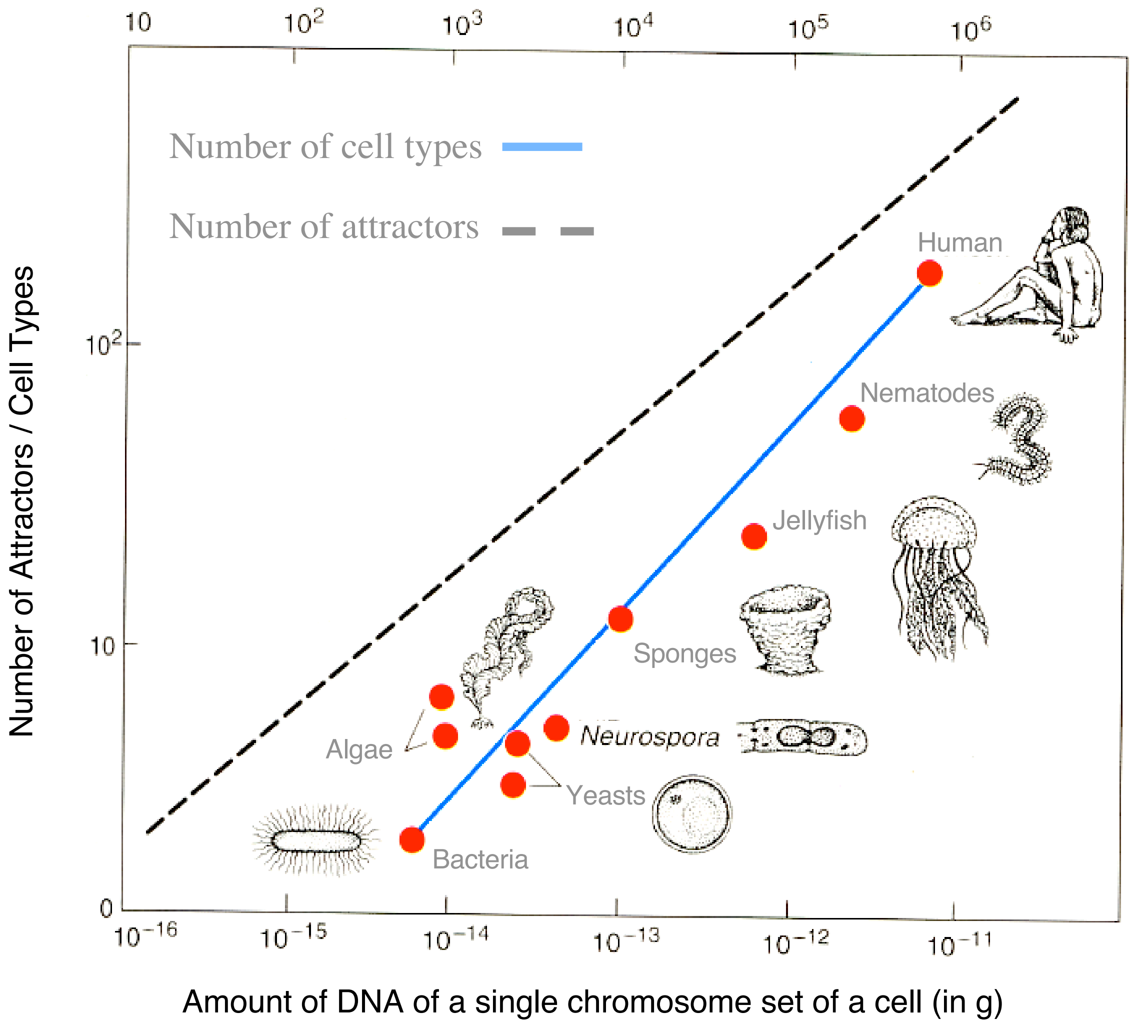
Attractor = Cell Type ?

- From the set of all possible gene activation patterns, the regulatory network selects a specific sequence of activations over time.

- A differenciated cell doesn't change its type any more.
  - Hence, only a constrained set of genes is active
  - = state cycle
  - = attractor?

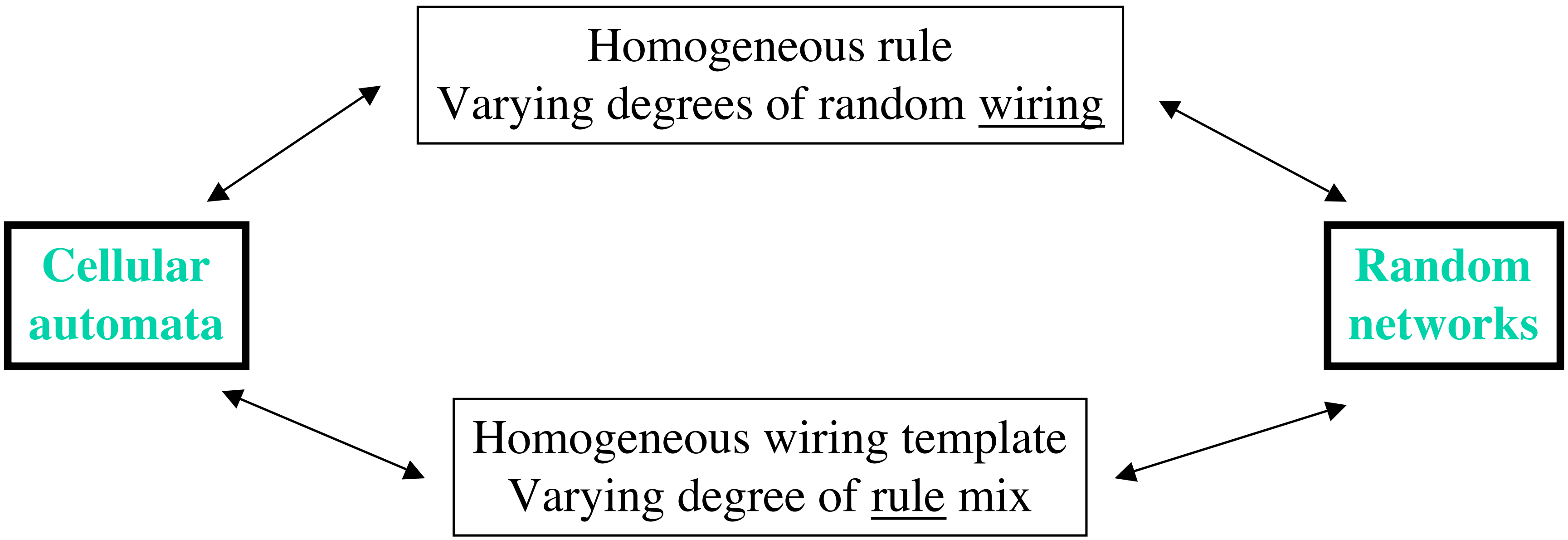
[S. Kauffman: Leben am Rande des Chaos]

Cell Types vs. Attractors

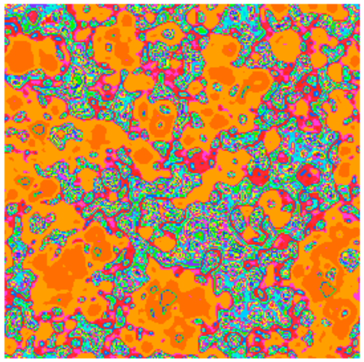


[S. Kauffman: Leben am Rande des Chaos]

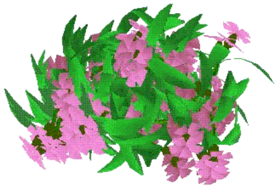
Intermediate Architectures



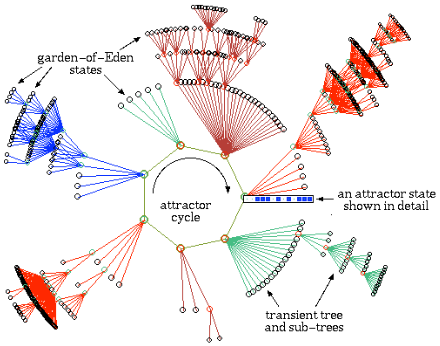
Cellular Automata



Lindenmayer Systems



Random Boolean Networks



Classifier Systems

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