Prologue

R. Scheidler and R. Woodrow

Richard Guy at Mount Assiniboine on his 90th Birthday, 2006.

We mourn the loss of Richard Kenneth Guy who passed away on March 9, 2020, at the impressive age of 103. Richard was a mathematical giant who made enormous and lasting contributions to our discipline. Active till the end, he was a distinguished researcher, a passionate educator, a generous philanthropist and an avid mountaineer. To us, he was also a valued colleague, mentor and friend.

This collection of contributions from colleagues and former students at the University of Calgary recounts some of Richard's involvement in research and mentorship, with special emphasis on the more than 13 years of his life after ninety. Richard's interests were too diverse and his contributions too numerous to do them all justice in one article. As a result, we chose to focus on three main areas of mathematics, drawing from experts in these fields for their recollections and perspectives on Richard's impact. A section on combinatorial game theory was provided by Richard's former doctoral student Richard Nowakowski. Richard's University of Calgary colleague Tibor (Ted) Bisztriczky supplied a section on geometry. A section on number theory was written by Richard's Calgary colleagues Mike Jacobson, Hugh Williams and the first author. Outreach and mentorship were equally important to Richard as research, and he has inspired many young scholars. We solicited input from two of his former University of Calgary protégés, Alex Fink and Julian Salazar, with whom he kept in contact until the last weeks of his life. We have also included a brief itemized biography and a short epilogue offering insight into other aspects of Richard's life both in and outside mathematics.

We thank all the contributors and are indebted to Claude Levesque (Université Laval) for providing the French translation of the English original. The photographs herein are courtesy of the University of Calgary, Ted Bisztriczky, Yanmei Fei, Jane Lancaster, Chic Scott, Hugh Williams and the first author.

This brief itemized biography highlights milestones in Richard's life. Much of it is drawn from the delightful book Young at Heart – The Inspirational Lives of Richard and Louise Guy (The Alpine Club of Canada, Canmore 2012) by Chic Scott, author, mountaineer and long-time friend of the Guys.

- Richard is born on September 30, 1916, in Nuneaton, Warwickshire (UK)
- BSc 1938 and MSc 1941 from Cambridge; Teacher diploma in 1939 from Birmingham
- Taught at Stockport Grammar School near Manchester 1939-1941
- Married Nancy Louise Thirian on December 21, 1940; three children (Elizabeth Anne, Michael, Peter)
- Meteorologist with the Royal Air Force (rank of Flight Lieutenant) 1941-1946; posted in Scotland, Iceland and Bermuda
- Taught again at Stockport Grammar School 1946-1947
- Taught at Goldsmith's College (a teacher training college) in London 1947-1951
- Faculty member at the University of Malaya, Singapore, 1951-1961
- Faculty member at IIT Delhi 1962-1965
- Faculty member at the University of Calgary 1965-1982, Professor Emeritus since 1982
- Honorary Doctor of Laws degree from University of Calgary 1991
- Richard and Louise Guy Lecture Series endowed as a gift from Louise to Richard for his 90th birthday, 2006
- Louise passes away at age 92 on September 30, 2010, on Richard's 94th birthday
- Richard and Louise Guy Hut in Yoho National Park opens its doors to overnight backcountry skiers, 2016
- City of Calgary Top 7 Over 70 Award and Immigrants of Distinction Lifetime Achievement Award, 2017
- Last "Climb for Wilderness" for the *Alberta Wilderness Association* on April 27, 2019
- Richard passes away on March 9, 2020

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Richard Guy and Game Theory

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R. Nowakowski

Richard K. Guy is chiefly responsible for the existence of Combinatorial Game Theory. Although he was not as prolific in game theory as in his other fields, he was a promoter behind the scenes and a mentor to many people.

Extending the Impartial Theory. Through his interest in chess, in 1947, Richard met T. R. Dawson who showed him a chess puzzle with pawns, now known as Dawson's Chess. Dawson proposed it as a misère problem (last player to move loses). Richard mis-remembered and solved the last-player-to-move-wins game. (This is a well-traveled path for starting a research career. As a 3rd year undergraduate, I misunderstood one of Richard's number theory homework problems. Richard turned my solution into my first research paper.) At that time, Richard didn't know about the work of Grundy or Sprague on impartial games. Independently, he went on to develop the theory. He was advised to contact C. A. B. Smith. Smith knew about the Sprague-Grundy theory and realized that Richard had shown that the theory was not just a curiosity but applied generally. Moreover, Richard had discovered octal games: essentially, the rules define what a player can remove from a heap and when the remainder can be split into two heaps. This class generated many intriguing conjectures and created combinatorial game theory as a research topic. Indeed, the most important conjecture—the sequence of values for every finite octal game is periodic—is still unsolved today. Richard was still pushing the boundaries of game theory at 90 [Fink and Guy 07].

Richard at work at Amiskwi Lodge near Golden, British Columbia, 1998

Assembling the Cast and Winning Ways. John H. Conway knew Richard's son Michael, who was also at Cambridge. Michael passed on to John all he knew about games. John was keen to learn more and a lifetime friendship and collaboration started. John asked about partizan games but it was many years before anyone had an answer. Elwyn Berlekamp had used the Guy-Smith paper [Guy and Smith 56] to further the analysis of Dots-and-Boxes. In 1967, Elwyn suggested that they write a book about games and Richard suggested John Conway be included. Winning Ways [Berlekamp et al 82] was finally published in 1982. This book is still as inspirational today as it was then and a must-read for any 'serious' student of combinatorial games. It contains many nuggets of wisdom, insights that have not been fully explored, and questions that direct research today. Of course, the book is not 'serious'. It contains much of Richard's (and John's) word play. Richard firmly believed that the right terminology and phraseology were important for motivation and to help people remember and understand concepts.

Promotion. After the publication of Winning Ways, Richard was involved in expounding the theory. In addition to innumerable talks, he organized and edited the Lecture Notes of the 1990 AMS Short Course on Combinatorial Games [Guy 92]. He helped organize the first MSRI and BIRS conferences on the subject. These led to the book series Games of No Chance which continues today. Richard wrote two of the first expository articles in the first book [Guy 96a, Guy 96b] and they are still well-worth reading. He also collated problems and wrote the first four Unsolved Problems in Combinatorial Game Theory articles for the series [Guy 96c]. A little known and hard-to-get gem is Richard's book Fair Game [Guy 89] which is an excellent introduction to impartial games.

Final Note. Richard K. Guy was great to be around. He was enthusiastic, always willing to roll up his sleeves and get stuck in. I owe my outlook on how and why to do mathematics, and the enjoyment I have obtained from my career, to him.

Richard Nowakowski, *Professor Emeritus in the Department of Mathematics and Statistics at Dalhousie University and a foremost expert on combinatorial game theory, obtained his PhD from the University of Calgary in 1978 under Richard Guy's supervision.*

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Richard Guy and Geometry

T. Bisztriczky

Richard Guy's research in Geometry was motivated by (1) the connections between elementary number theory and geometry, and (2) the many geometrical problems that are intuitive (in the sense of easy to state) or appealing to students and teachers (in math camps and competitions). His contributions to the field follow the style of such British geometers as D.M. Sommerville and H.F. Baker. The latter is best known to us via his six volume Principles of Geometry [Baker 10] and An Introduction to Plane Geometry [Baker 71]

As examples of (1), we have *The Lighthouse Theorem, Morley & Malfatti — a budget of paradoxes* [Guy 07] and *Triangle-rectangle pairs with a common area and a common perimeter* [Bremner and Guy 06]. In the former, Richard notes that "the combination of geometry and number theory is dear to my heart", and the combination here is between integer-edge triangles and primes $p > 7$ with the property that $p = 3n+1$ and $p^6 = a^2+4762800b^2$ for unique integers $|a|$ and $|b|$. In the latter, he and Andrew Bremner show that such triangle-rectangle pairs are parametrized by a family of elliptic curves.

Special Session on Discrete Geometry and Convexity, Joint Math Meetings, Atlanta 2017

Regarding (2), we refer to Richard's many contributions to the Problem Sections of the A.M.Monthly and the Math. Magazine, and to his book with H. Croft and K. Falconer, *Unsolved Problems in Geometry* [Croft et al 94]. As W. Moser foretold in his AMS review of the text [Moser 94], the volume became a sourcebook for anyone wishing to do research in intuitive (convex, discrete and combinatorial) geometry.

Richard K. Guy was an ideal colleague: very knowledgeable, always supportive and unfailingly kind. With his office door always open and his ever willingness to provide counsel and exchange ideas, he was very much an epitome of the cinematic venerable professor. We are grateful for the many decades that he was with us.

Tibor (Ted) Bisztriczky *is a Faculty Professor and a Professor Emeritus in the Department of Mathematics and Statistics at the University of Calgary. His research interests include convex and discrete geometry, particularly the study of polytopes. He and Richard were colleagues for over forty years, and shared a corridor for the last thirty.*

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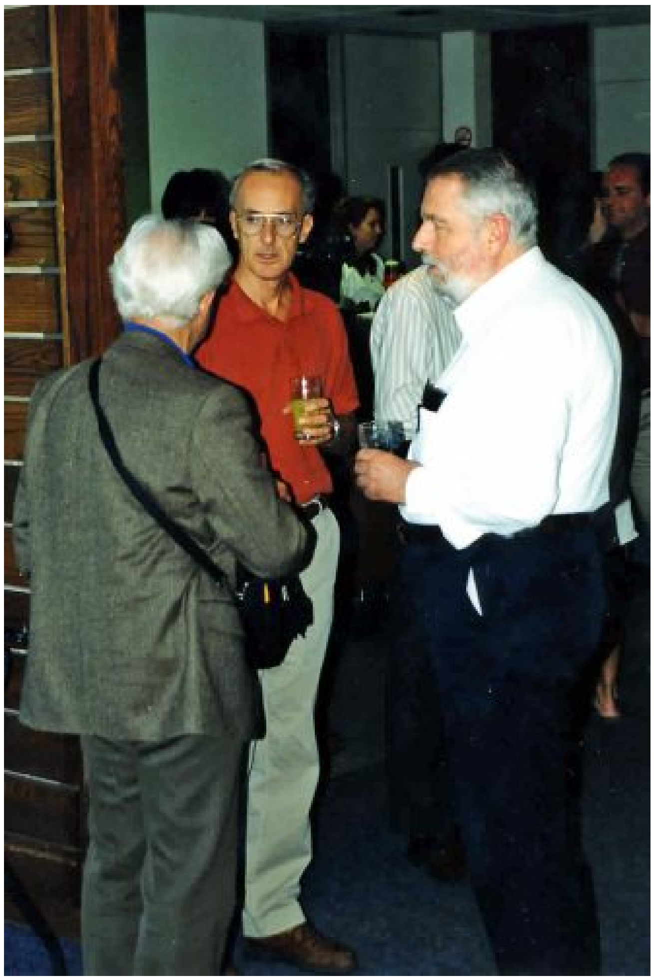
M. J. Jacobson Jr., R. Scheidler and H. C. Williams

Even as a small child Richard Guy was fascinated by numbers. When he was 17, he purchased a copy of Dickson's encyclopaedic *History of the Theory of Numbers* [Dickson 66] and fell under its spell. The cost of this book at the time was 6 guineas, a lot of money—more than he paid for his Master's degree from Cambridge. Dickson's history continued to exert a strong influence on Richard throughout his academic life. He published his first significant paper in number theory in 1958 [Guy 58]. Arguably, no work of Richard's exemplifies his love of numbers more than his wonderful *Book of Numbers*, joint with John H. Conway [Conway and Guy 96]. Richard's substantial volume of published works in number theory include some 50 coauthors, among them Elwin Berlekamp, John Conway, Paul Erdös, Derrick Lehmer, Yuri Matiyasevich, Alexander Oppenheim, John Selfridge, and Daniel Shanks. His predominant number theoretic interest was in integer sequences in any form or context, including their appearance in combinatorics, geometry and Diophantine problems. Richard's contributions to the field are too numerous to allow a complete account here, so we only provide some examples of his work, with special attention to research he conducted in his late 90s and beyond.



Richard, Andrew Bremner and John Selfridge at CNTA-VI, Winnipeg, 2006

> *Arguably, no work of Richard's exemplifies his love of numbers more than his wonderful Book of Numbers, joint with John H. Conway*

Tweet

Aliquot sequences were a particular life-long passion of Richard's. These are iterates n, $s(n)$, $s(s(n))$, … , $s^{(k)}(n)$ of the sum of proper (or aliquot) divisors function $s(n) = \sigma(n) - n$, for n a positive integer. Catalan [Catalan 88], later corrected by Dickson [Dickson 13], conjectured that all aliquot sequences terminate or become periodic and are hence bounded. The smallest integer for which this conjecture remains unsettled is $n = 276$; as of the time of writing this article, its aliquot sequence has been computed to 2139 terms and the 2140th term is composite and known to have a(n as yet unfactored) divisor of 213 decimal digits. In a series of reports published in the 1970s, Richard and John Selfridge discovered that under certain conditions aliquot sequences can become quite long. This led them to propose a counter-conjecture in 1975 [Guy and Selfridge 75] that many, perhaps almost all, sequences for even n diverge. The question of which conjecture is correct remains unresolved.

Richard was keenly interested in evangelizing his conjecture and continued to work on it to the end. A 2012 result of Bosma and Kane [Bosma and Kane 12] shows that the geometric mean (over all n) of the *amplification* $s(2n)/2n$ is less than 1, thus suggesting that the terms of an aliquot sequence tend to decrease on average. Richard believed that this finding does not capture the true nature of aliquot sequences because it does not take into account how frequently (if ever) an integer occurs as a value $s(2n)$, and it fails to account for the *guides* and *drivers* described in [Guy and Selfridge 75]. These are certain divisors of $s^{(k)}(n)$ that persist from one term to another with high probability and that in almost all cases cause the sequence to increase. Indeed, Pomerance [Pomerance 18] showed that the geometric mean of $s(n)/n$ for $n \equiv 2 \pmod 4$ is less than 1, whereas it exceeds 1 for $n \equiv 0 \pmod 4$.

With further analytic results seemingly out of reach for the time being, Richard instead turned to a quest for stronger numerical evidence supporting his point of view. Mike Jacobson recounts how Richard began to subtly recruit him to join in this cause, beginning with an e-mail with the rather cryptic subject line "Would you like to factor a number?" The number was of course factored, and when asked what this was about, Richard gladly offered an explanation that began as follows:

> *I'm calculating an aliquot sequence. This is rather a lost cause, but it has grabbed me from even before a bright young undergrad named Jeff Lagarias was introduced to me by Danny Kleitman long years ago at MIT. Selfridge and I devoted thousands of hours on two Olivettis, and Mike Williams used a more sophisticated machine down in the basement of this building [the Mathematical Sciences building at the University of Calgary], when it was only four stories high.*

Jacobson describes how at any time, Richard's office computer showed at least one open window actively computing terms of some aliquot sequence and performing the necessary factorizations. At the time of the initial request to Jacobson, Richard was using Pari/GP to manually iterate the aliquot sequence for $n = 99225$ because, again in his words, "the smallest odd number which shows any signs of getting to infinity is 99225". He had extended this sequence to well over 700 terms and regularly needed to factor integers of over 100 decimal digits. Jacobson eventually automated the factorization process for Richard and helped him extend the sequence to more than 3400 terms, which led to an even more ambitious project. In 1976, Richard had written a survey of then state-of-the-art integer factorization methods [Guy 76] that became very influential with the advent of the RSA cryptosystem in 1978. Following earlier computations undertaken by Richard's former Master's student Stan Devitt in 1976 [Devitt 76] that were surely inspired by Richard's survey, Richard, Jacobson and then Calgary students Kevin Chum and Anton Mosunov performed extensive computations of the geometric mean of $s(n)/n$ by modeling an aliquot sequence as a Markov chain [Chum et. al. 18]. To Richard's delight, along with a variety of other related numerical results capturing data of actual aliquot sequences, these computations provide empirical evidence that the geometric mean of $s(n)/n$ in fact exceeds 1, exactly as he had hoped and predicted. Work on the Guy-Selfridge conjecture by Jacobson and his students is ongoing.

Richard was fond of finding arrangements of numbers subject to certain constraints governing neighbour relationships. He was intrigued by the simplicity of such questions and the frequent immense difficulty of proving even the simplest existence or counting results on such arrangements. Richard was convinced that for all sufficiently large n, there exist permutations of the integers 1, 2, … , n such that any two adjacent entries sum to a square, cube, triangular or pentagonal number, or any "reasonable" polynomial in n. The square case was only recently settled by R. Gerbitz [Gerbitz 2018] who established an affirmative answer for all $n \geq 25$ ($n \geq 32$ for circular arrangements). All other cases remain wide open. In a delightful manuscript entitled "Fibonacci plays Billiards" [Berlekamp and Guy 03], Elwin Berlekamp and Richard gave a complete characterization of values n that admit permutations of the first n positive integers such that any two neighbours sum to a Fibonacci or Lucas number. The title stems from a methodology that facilitates the search for number arrangements by placing the numbers 1, 2, … , n on the perimeter of a billiard table and considering paths of billiard balls as they bounce off the corresponding points on its cushions at a 45 degree angle. In the summer of 2017, then centenarian Richard, with help from Calgary colleague Renate Scheidler, recruited Ethan White, an undergraduate student at the time, to look into analogous problems where sums are replaced by absolute differences. Eventually, they settled on the question of circular arrangements where the absolute difference of any two adjacent terms takes on one of two fixed given values a or b. Aided by White's computations, they employed Richard's number wall [Conway and Guy 96, pp. 85-89] to try to discover linear recurrences for the counts $N_{a,b}(n)$ of such arrangements of length n. They found explicit recurrence relations for the pairs $(a,b) = (1,2)$, $(1,3)$, $(2,3)$, $(1,4)$ and eventually employed the graph theoretic transfer matrix method to prove that $N_{a,b}(n)$ satisfies a linear recurrence relation whenever such arrangements exist for a given pair (a,b) [White et al 20].

Richard was also interested in Diophantine problems, particularly the question of whether integers could be represented by certain types of equations. A *Diophantine equation* is an equation for which solutions are restricted to the integers or rational numbers; for example, $(x,y) = (8,3)$ is a solution to the Diophantine equation $x^2 - 7y^2 = 1$. Richard began a long-term collaboration in this area with Andrew Bremner in the late 1980s that lasted more than 15 years. In 1993 Bremner, Richard and Richard Nowakowski settled the question, first posed by Melvyn J. Knight, of which integers n can be represented in the form

$$n = (x+y+z)(1/x+1/y+1/z),$$

with integers x, y, z [Bremner et al 93]. For example, for $n = 62$, we have the solution $x = 5075$, $y = 128050$, $z = 160602$. They found that this question reduces to the problem of finding integer points on a certain elliptic curve with rational 2-torsion and computed the Mordell-Weil rank of this curve for all n with $|n| \leq 1000$.

Integer sequences in the context of geometry also appealed to Richard. An example of a question of this flavour is the problem of tiling a $4\times(n-1)$ rectangle with dominos (1×2 tiles). Richard knew that the sequence $(A_n)_{n\geq 0}$ representing the number of distinct such tilings satisfies the fourth order linear recurrence

$$A_k = A_{k-1} + 5A_{k-2} + A_{k-3} - A_{k-4}$$

with $A_0 = 0$, $A_1 = 1$, $A_2 = 1$, $A_3 = 5$, $A_4 = 11$, $A_5 = 36$, etc. He noticed that the sequence $(A_n)_{n\geq 0}$ seemed to be a divisibility sequence (A_n divides A_m whenever n divides m). This observation led to a collaboration with Hugh Williams and his former doctoral student Eric Roettger that produced a series of papers [Roettger et al 13, Roettger et al 15, Williams and Guy 15], culminating with a solution to Lucas' unsolved problem of generalizing the Lucas sequences to the setting of higher order recurrences.

Over several decades, somewhat simultaneously with the conception and writing of John Conway's renowned *Triangle Book* [Conway and Sigur 15], Richard compiled and proved a comprehensive body of results in a monograph simply entitled *The Triangle* [Guy 20]. In addition to a wealth of number theoretic and geometric facts about triangles, this 240 page work contains a collection of exquisite figures, all meticulously produced through Richard's wizard mastery of LaTeX. Richard was captivated in particular by the following construction, explained and beautifully illustrated on pp. 43 ff [Guy 20]. For a triangle *ABC*, take any point *P* on its circumcircle and reflect it on the edge *BC* to obtain a point *A*' that defines a new triangle *A'BC*. Intersect the perpendicular to the edge *BC* with the circumcircle of this new triangle to obtain a point *P*'. Similarly, reflect *P* on the edges *AB* and *BC* to obtain triangles *AB'C* and *ABC*' and points *Q*', *R*'. The three points *P*', *Q*', *R*' lie on a *Steiner line* parallel to the *Wallace line* of *P* and twice its distance to *P*. Repeat the entire process starting with *P*', *Q*', *R*' to generate 9 further points etc. Richard likened this construction to computing scalar multiples of a given fixed point on an elliptic curve and was curious about the behaviour of this *trisequence*, particularly the possibility of periodicity. Richard credits Andrew Bremner with the discovery of four 3-cycles and subsequently Alex Fink, whom he mentored during Alex's undergraduate years at Calgary, for observing that every starting point *P* leads to three 6-cycles.

The University of Calgary's Department of Mathematics and Statistics celebrates Richard's 100th birthday, 2016

Richard had a phenomenal gift for pattern recognition and an uncanny ability of separating the grain of beautiful number theoretic structure from the chaff of coincidental similarities. In the course of his investigations of various sequences, Richard discovered what he wittily referred to as "The Strong Law of Small Numbers". In his very engaging and influential paper of the same title [Guy 88], he discussed 35 examples of patterns that seem to appear when we check small values of *n*. Some work, but many don't. He concluded that there aren't enough small numbers to meet the many demands made of them. He followed this paper two years later with his second law [Guy 90] which states "When two numbers look equal it ain't necessarily so." Both these papers should be required reading by any graduate student of mathematics.

One of Richard's most lasting contributions to the field is his monograph *Unsolved Problems in Number Theory* [Guy 04]. A marvellous compilation of number theoretic problems and commentary that has gone through three editions and is instantly infectious, this remarkable book has stimulated generations of aspiring number theorists, several of whom have gone on to have stellar careers, and continues to be a source of inspiration for scholars and in the field.

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Hugh C. Williams *is a Professor Emeritus in the Department of Mathematics and Statistics and the former iCORE Chair in Algorithmic Number Theory and Cryptography at the University of Calgary as well as Professor Emeritus in the Department of Computer Science at the University of Manitoba. His research interests include computational number theory, cryptography and the history of mathematics and computation.*

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R. Scheidler, based in part on accounts by A. Fink and J. Salazar

Richard Guy was a dedicated educator and mentor to students of all ages. He considered his efforts in this area at least as important as his research contributions. He supervised graduate students until 2002, when he was 86, and undergraduates until the age of 101. Even as a centenarian, Richard participated in the weekly gathering of the Calgary number theory faculty and students, where he would pose problems, ranging from challenging to recreational, and gently but inevitably tempt at least one student into pursuing the problem further or crunching some numbers for him.

Until well into his 90s, Richard was a regular at Wednesday's Calgary *Math Nites*, a weekly enrichment program where faculty members and graduate students expose grade 7-10 students to mathematical discovery and engage them in problem solving. It was there that Richard met two of his most successful charges who kept in contact and collaborated with him until the end: Alex Fink, now a faculty member at Queen Mary University of London, and Julian Salazar, who obtained a BA in Mathematics and a secondary concentration in computer science from Harvard in 2017 and went on to embark on a career in machine learning with Amazon.

Alex Fink attended *Math Nites* starting in grade 4. While in high school, Richard invited him to attend his reading course in combinatorial game theory. Fink went on to train with Richard for the Putnam exam and, supported by two NSERC USRAs, conduct research under Richard' supervision. After completing his undergraduate education at the University of Calgary, he continued to keep in touch with Richard in person and online. "There was a lot to finish at that point", Fink comments. Between 2006 and 2017, he and Richard co-authored three research papers [Fink and Guy 07, Fink et al 08, Fink and Guy 17] and two expository articles [Fink et al 06, Fink and Guy 09]. Fink paraphrases some of the lessons learned from Richard early on:

> *"Have multiple balls in the air: it's good to have somewhere to turn when you hit a wall on project A, and your subconscious will be chipping away at project B in the meantime anyhow."*
>
> *"Write it all down. The easiest way to edit is to cut things out."*
>
> *"Go to conferences even before you have the background: it won't be a waste, you'll absorb some of the language and be better prepared for the next one."*
>
> *"Be careful of making (even implicit) assumptions that might alienate some of your audience. Hence never 'well-known', always 'well-known to those who well know it'."*

Richard with two University of Calgary graduate students, 2003

Julian Salazar recalls feeling welcome from Richard's first e-mail reply and describes their meetings as socratic. "He [Richard] just patiently described what he was thinking about, I'd ask questions, he'd ask them back. After 1-2 years of casual chats, he asked a question which I proved (our Theorem 7) on the train home. That moment, of devising something new, has defined much of my adult life." The work Salazar refers to is [Guy et al 14], published when he was 20 and Richard was 98. Richard provided financial support for Salazar to attend and present at MathFest and took him for dinner with Noam Elkies who later became Salazar's senior thesis advisor at Harvard. Salazar notes that Richard "pursued problems because they were interesting; not because they were technically challenging or trendy" and credits Richard with the lesson to "do what you enjoy, independent of the credentials or the default path".

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Richard and Louise at Assiniboine Lodge in British Columbia on Richard's 90th birthday

While this collection of contributions focused on Richard's years as a mathematician at the University of Calgary, it would be amiss not to include a few observations about other aspects of his rich and multi-faceted life. Richard's other great love, besides mathematics, was Louise, his wife of some 70 years. Richard and Louise were pioneers in many ways. In the 1950s and 60s, at a time when — contrary to today — academic career moves across countries and continents were highly unusual, they spent 13 years in Singapore and India where Richard taught university mathematics. Louise and Richard were outspoken pacifists during Cold War times when such sentiments were not always welcome. They were ardent mountaineers and shared a passion for nature and conservation long before the modern environmental movement received traction. Many of the much younger colleagues hired during Richard's tenure as Head of Calgary's Department of Mathematics and Statistics recall back-country hikes in the Canadian Rockies where they would see Louise and Richard zip past them on the trail.

Richard and Louise were also generous philanthropists who supported many causes in service of mathematics and life outdoors. They were life-long active members of the Alpine Club of Canada and the Calgary Mountain Club. In 2016, a backcountry ski hut in Yoho National Park named in honour of the Guys opened its doors to backcountry skiers, thanks in large part to a substantial donation by Richard in memory of Louise. Their participation in the annual Calgary Tower climbs to raise funds for the Alberta Wilderness Association, continued by Richard on his own until the age of 102, carrying along a photo of Louise after her death, are legendary in Calgary and among the Alberta outdoors community. During the 1980s, through the dedication and generosity of the Guys, the University of Calgary acquired the Eugene Strens Recreational Mathematics Collection of books, periodicals, puzzles and manuscripts dating from the 17th to the 20th century. In 2006, as a gift to Richard for his 90th birthday, Louise endowed the Richard and Louse Guy Lecture Series, featuring annual public talks by an impressive collection of the world's finest mathematicians.

A memorial scholarship in Richard's name was endowed by his friends and colleagues in March 2020. We leave the reader with some links to what others have said about Richard's life in and outside mathematics.

- Richard's obituary in the Calgary Herald
- University of Calgary remembers Richard Guy
- Chic Scott's chronicle of Richard's life for the Alpine Club of Canada
- Top 7 over 70 – Richard's last Calgary Tower Climb 2019
- Antony Bonato's 2017 interview of Richard
- Richard and Louise Guy Lecture Series
- Strens Recreational Mathematics Collection
- Dr. Richard Guy Memorial Scholarship in Mathematics