Digital Signal Processing Introduction

CPSC 501: Advanced Programming Techniques
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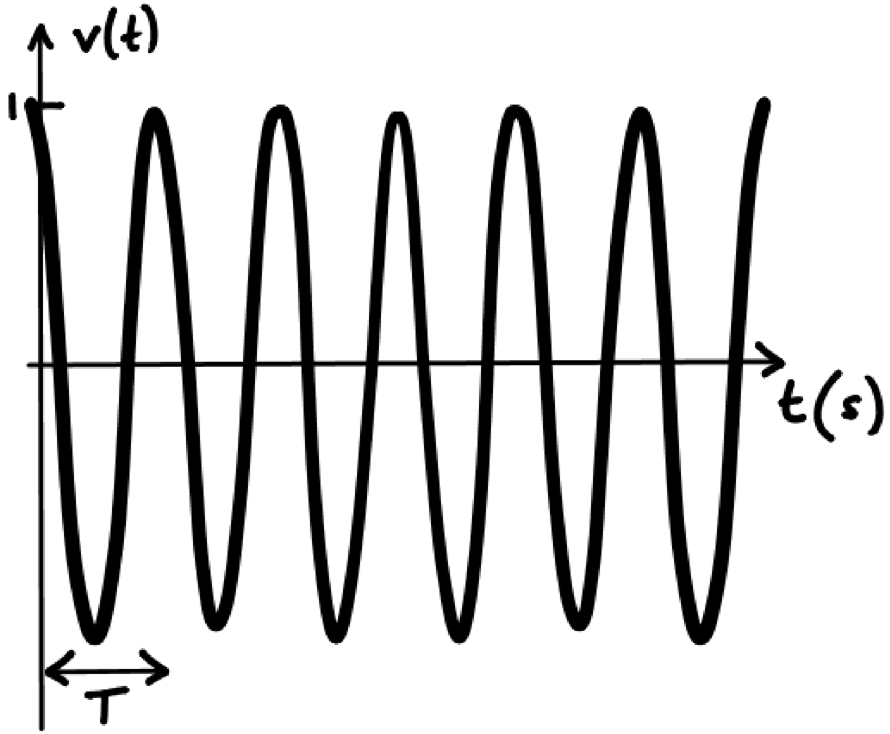
Signals

How to we get a signal

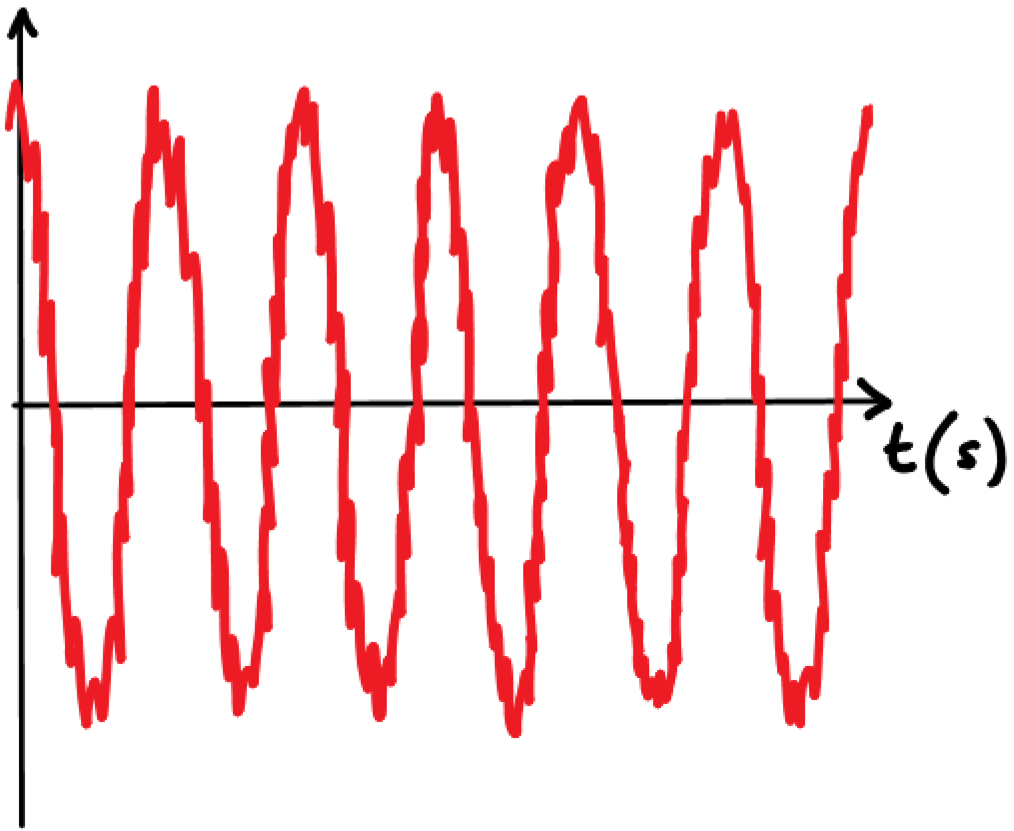
Analog Signal

- Analog signal. This signal $v(t)=\cos(2\pi ft)$ could be a perfect analog recording of a pure tone of frequency $f=1$ Hz.
- The period $T=1/f$ is the duration of one full oscillation.



Noisy Signal

- Noisy analog signal. Noise degrades the sinusoidal signal.
- It is often impossible to recover the original signal exactly from the noisy version



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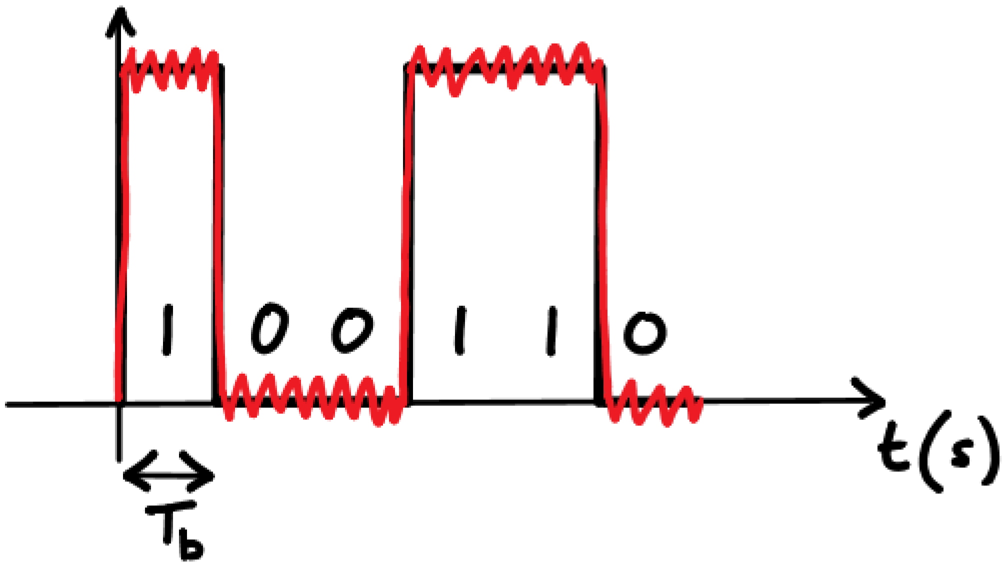
Digital Signal

- Analog transmission of a digital signal.
- Consider a digital signal 100110 converted to an analog signal for radio transmission.
- The received signal suffers from noise, but given sufficient bit duration T_b, it is still easy to read off the original sequence 100110 perfectly.



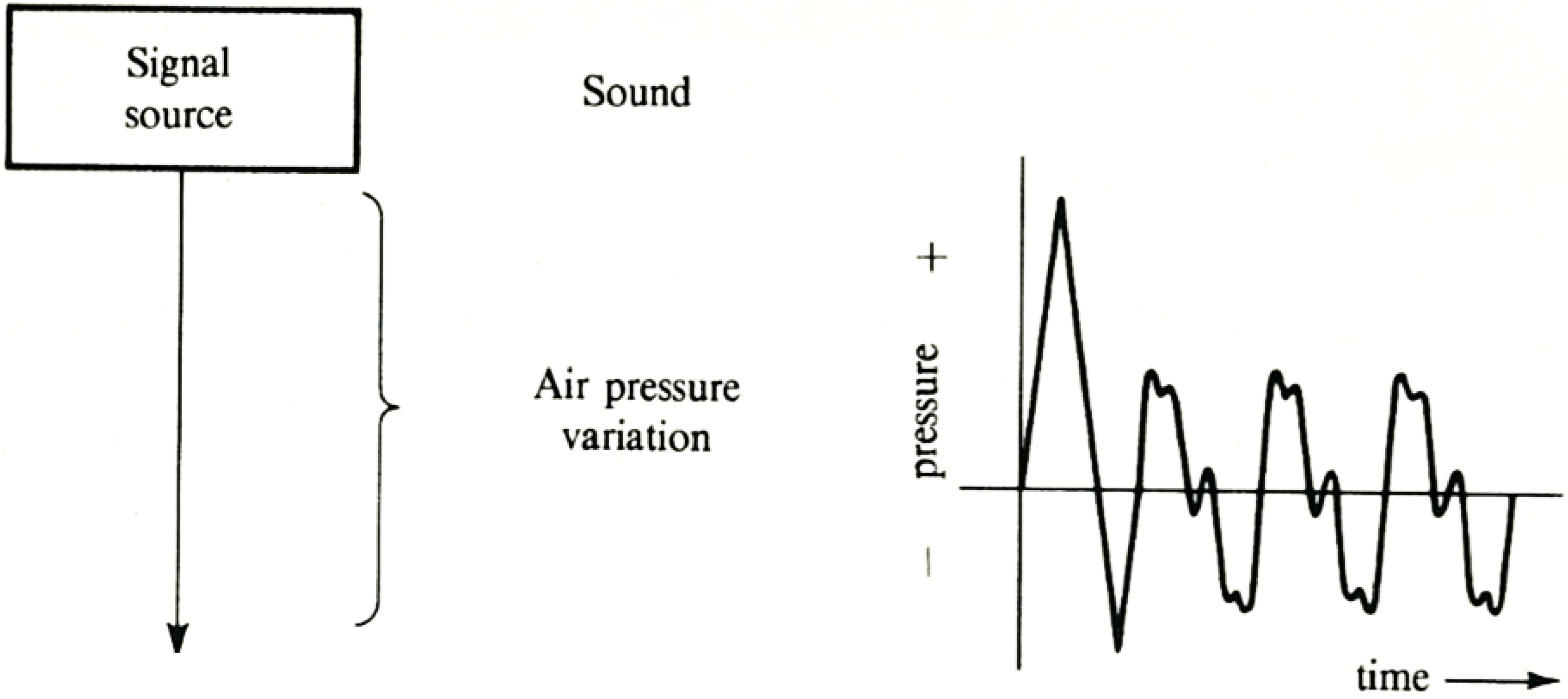
Sampling

- A continuous signal may be sampled
  - i.e. measured periodically at small intervals of time, and converted into a series of numbers (samples)
    - Such a series is a digital signal
  - An **analog-to-digital converter** does the sampling
    - E.g. Sampling an audio signal

From Voice to Bits

How to we get a signal

Original Signal



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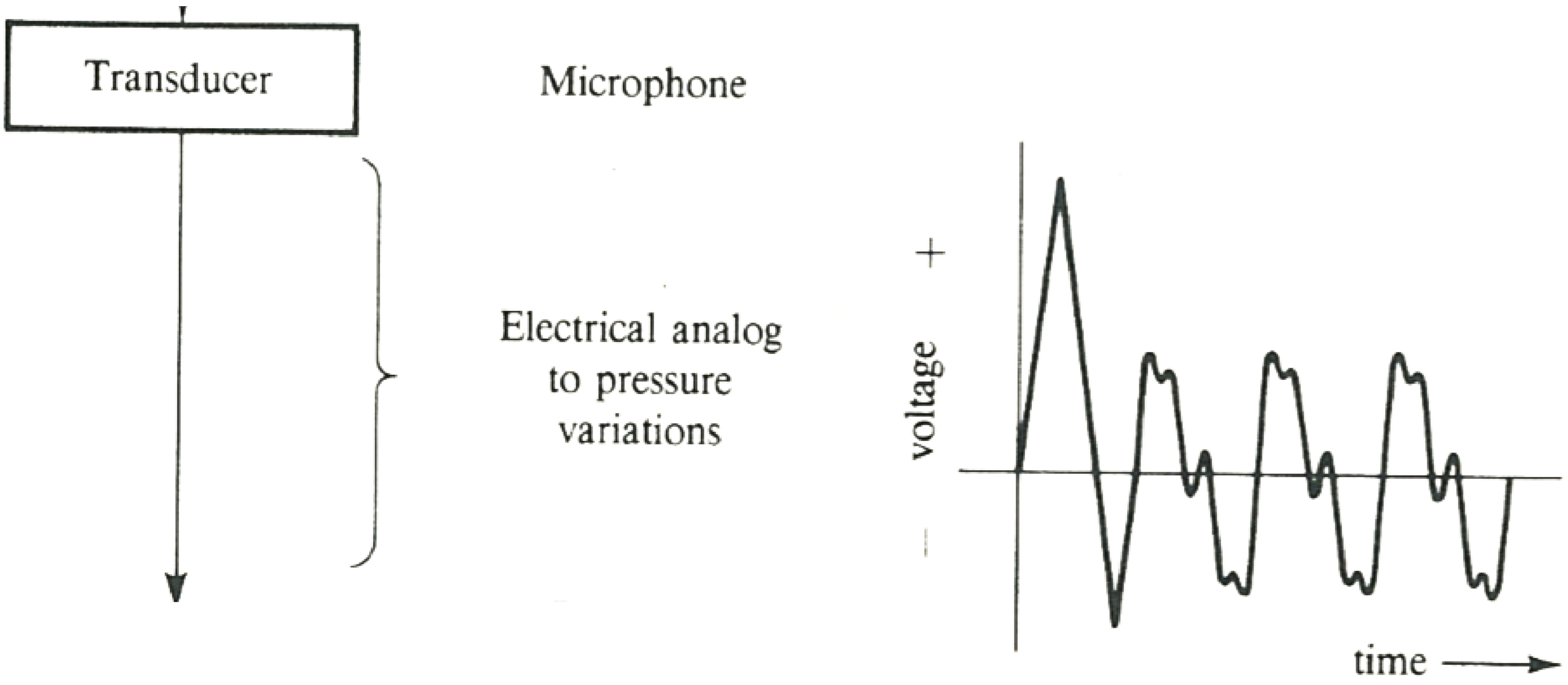
Analog Signal: Via Transducer



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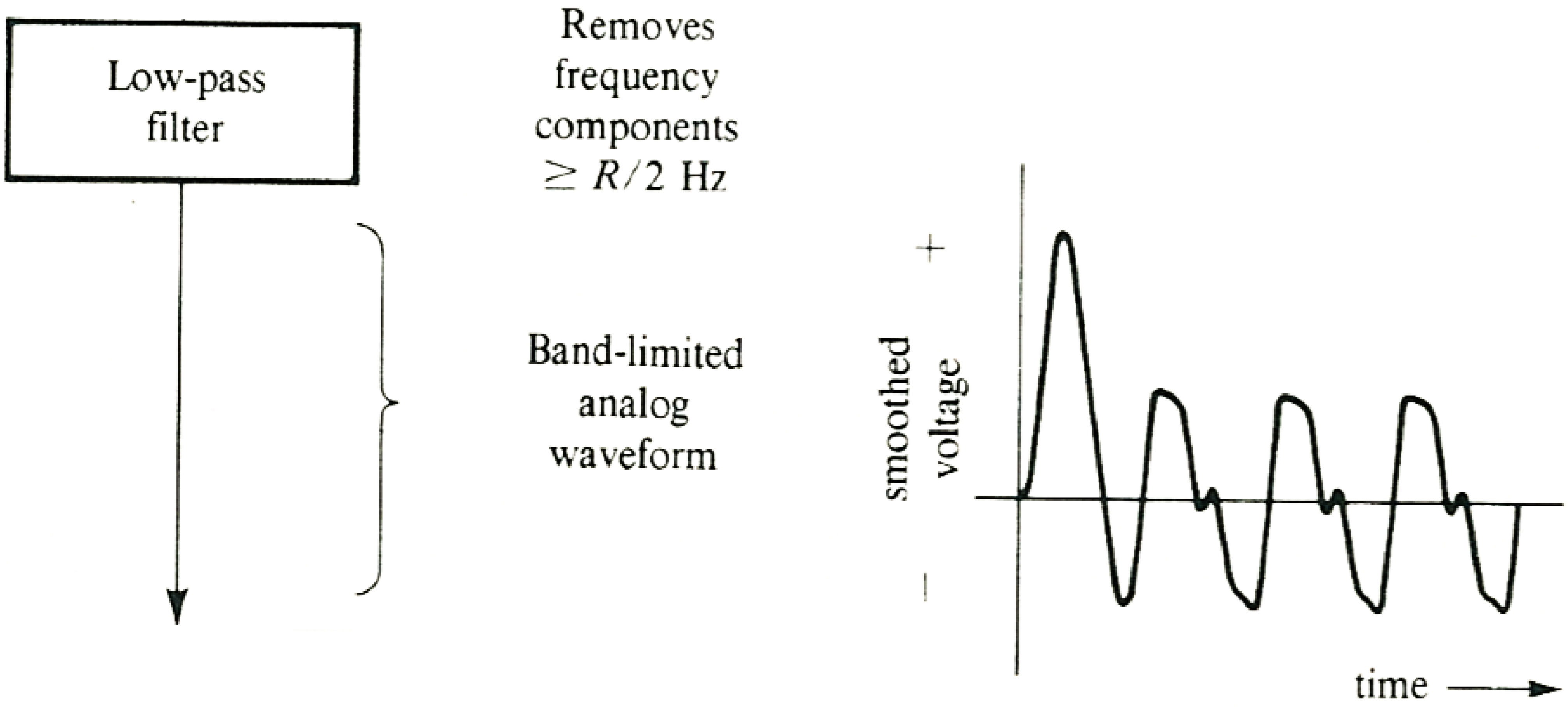
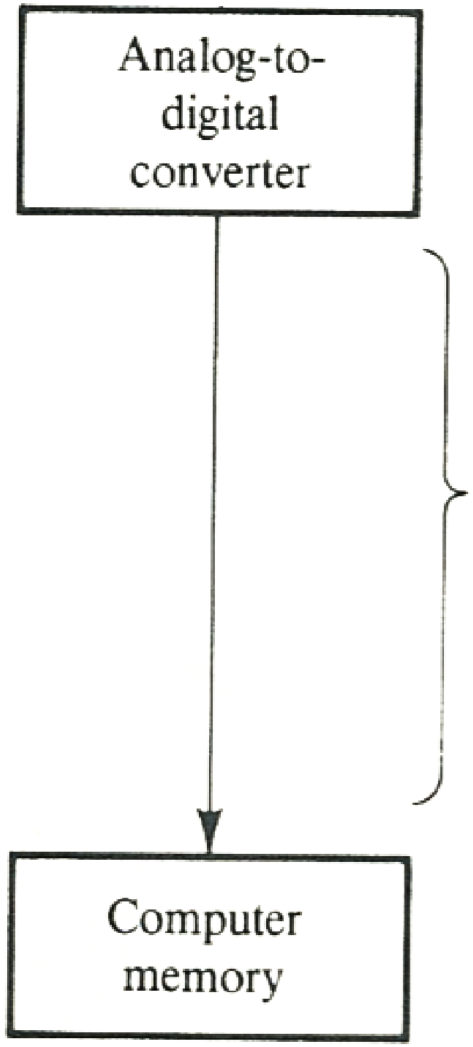
Analog Signal Cleaning



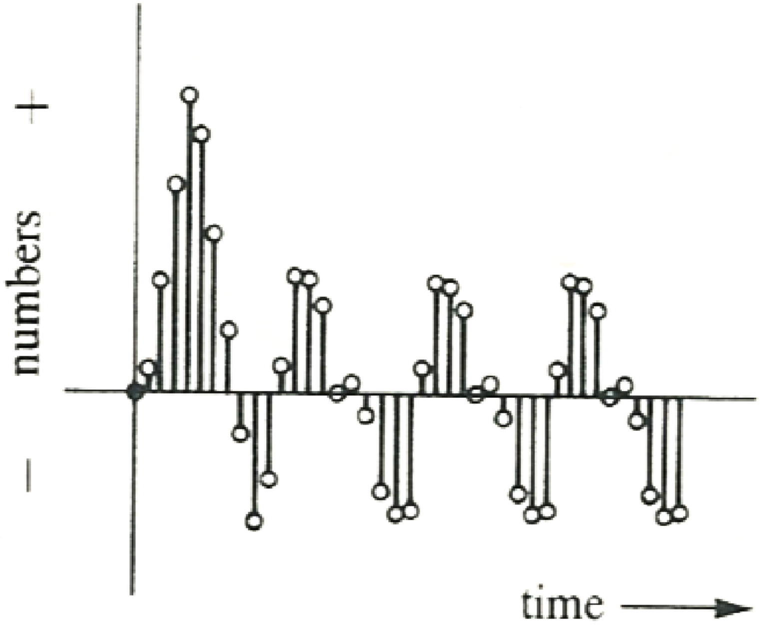
Analog to Digital



Samples at R Hz and quantizes to B bits

Stores complete representation as sequence of binary numbers

Discrete representation of band-limited analog waveform (digital signal)



And back again

Sampling and Quantization

- A **digital-to-analog converter** converts the digital signal back into an analog signal

Sampling

Sampling

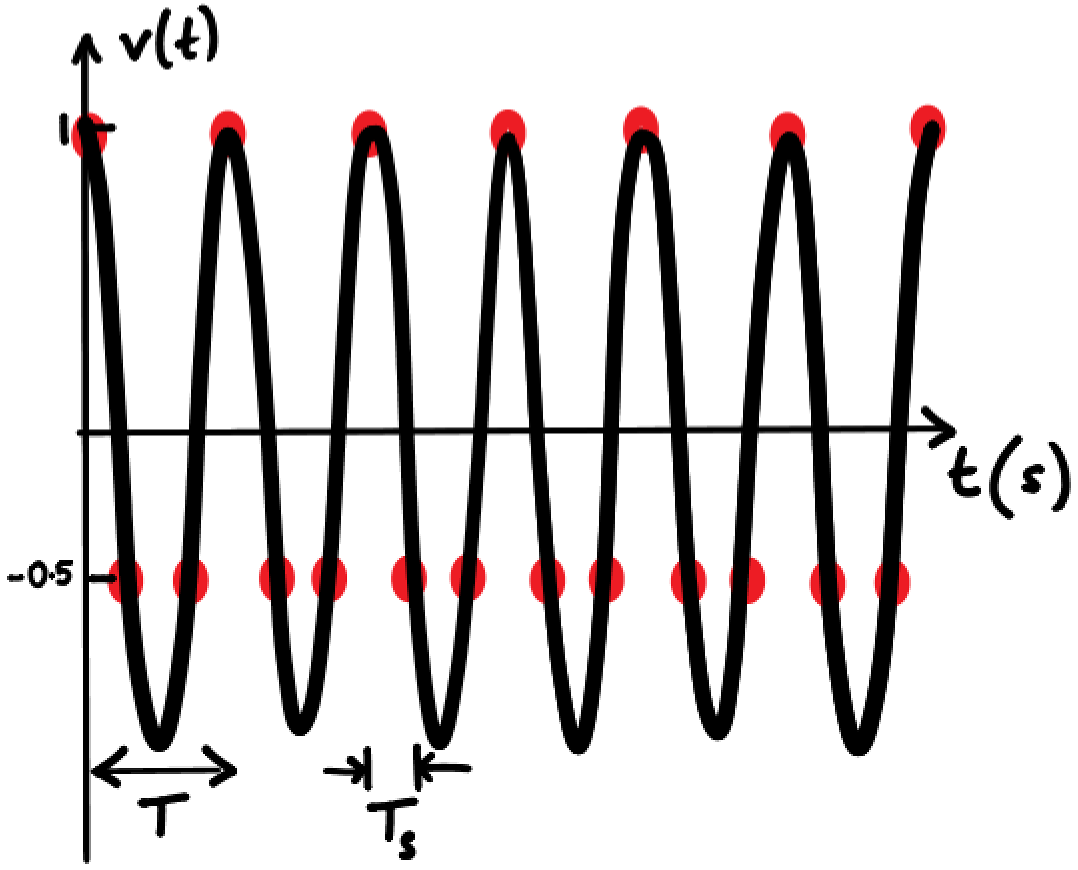
- Sampling is the process of recording an analog signal at regular discrete moments of time.

- The sampling rate f_s is the number of samples per second.

- The time interval between samples is called the sampling interval $T_s = \frac{1}{f_s}$.

Original signal

- The signal $v(t)=\cos(2\pi ft)$ is sampled uniformly with 3 sampling intervals within each signal period T.
- Therefore, the sampling interval T/3 and the sampling rate 3f.
- Notice that there are three samples in every signal period T.



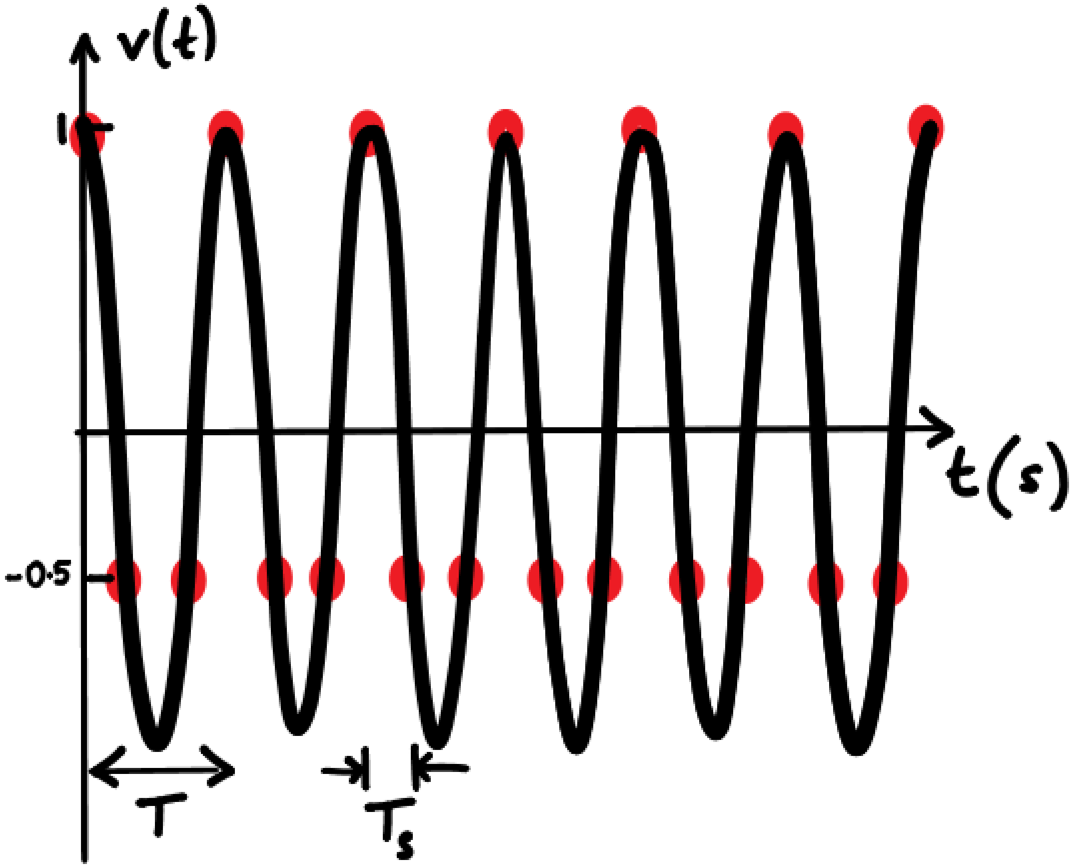
Sample points

- To express the samples of the analog signal x(t), we will use the notation x[n] for example
  - integer values of n index the samples
- Typically, the n=0 sample is taken from t=0
- Consequently, the n=1 sample must come from the $t = T_s$ time point, exactly one sampling interval later; and so on.
- sequence of samples can be written as

$$x[0] = x(0), x[1] = x(T_s), x[2] = x(2T_s), \ldots$$

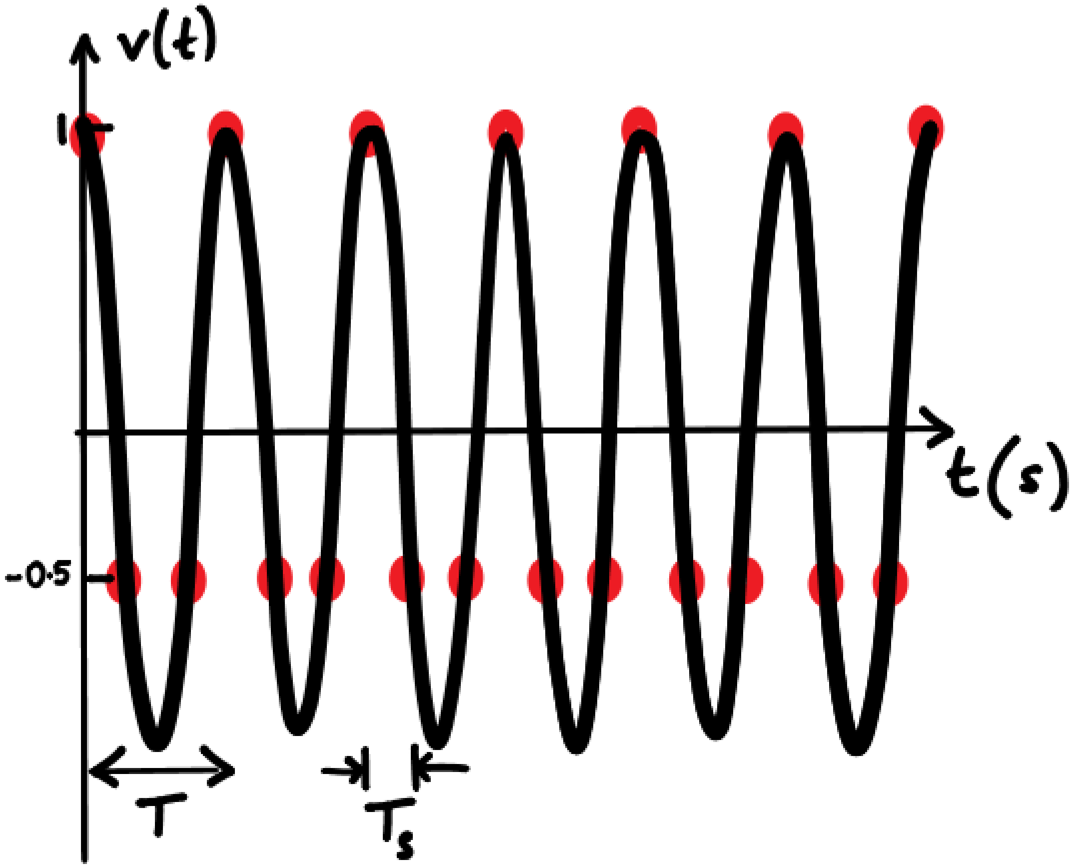
Store sample: extracted from formula

- $x[n] = x(nT_s)$ for integer n
- Our signal was
- $x(t) = \cos(2\pi f t)$
- $x[n] = \cos(2\pi f n T_s)$
- $x[n] = \cos(2\pi f n \frac{T}{3})$with $T_s = \frac{T}{3}$
- $x[n] = \cos(\frac{2\pi n}{3})$ as $\mathrm{T} = \frac{1}{f}$



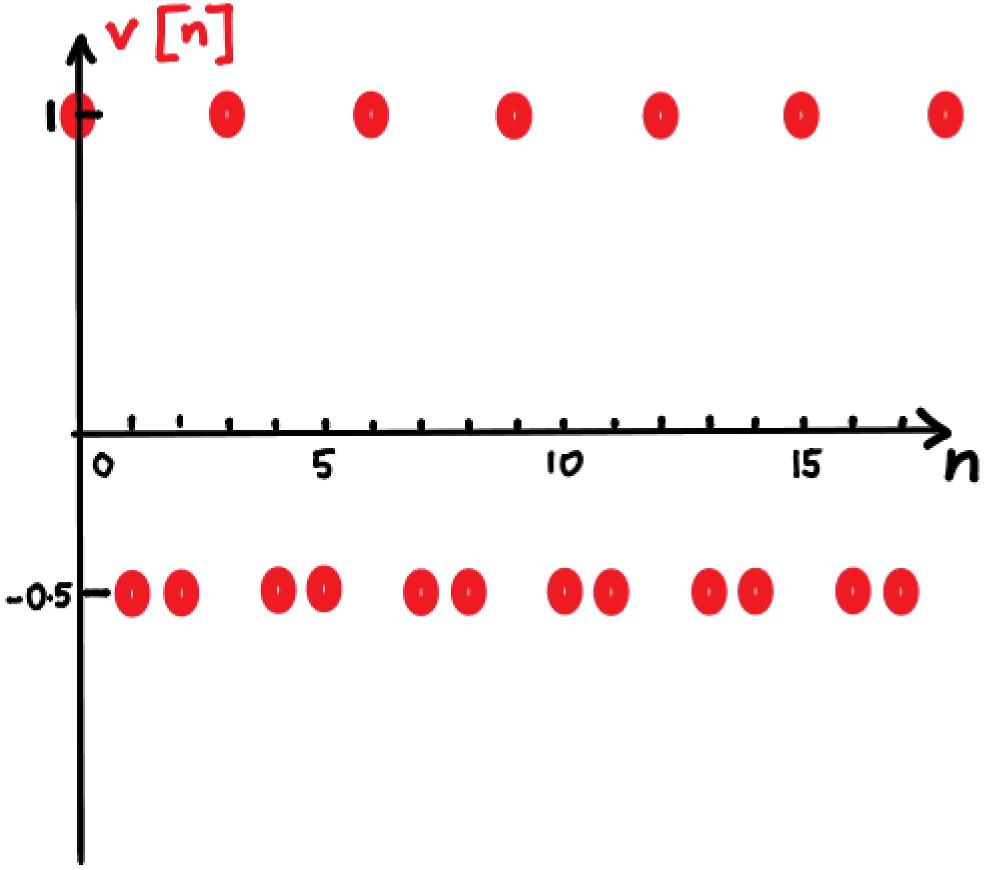
Stored sample: if measured

- $x[n] = \cos(\frac{2\pi n}{3})$
- $x[0] = \cos(0) = 1$
- $x[1] = \cos\left(\frac{2\pi}{3}\right)$ =-0.5
- $x[2] = \cos\left(\frac{4\pi}{3}\right)$ =-0.5
- $x[3] = \cos(2\pi) = 1$



Can we rebuild it?

- $x[n] = \cos(\frac{2\pi n}{3})$
- $x[0] = \cos(0) = 1$
- $x[1] = \cos\left(\frac{2\pi}{3}\right)$ =-0.5
- $x[2] = \cos\left(\frac{4\pi}{3}\right)$ =-0.5
- $x[3] = \cos(2\pi) = 1$



Sampling rate

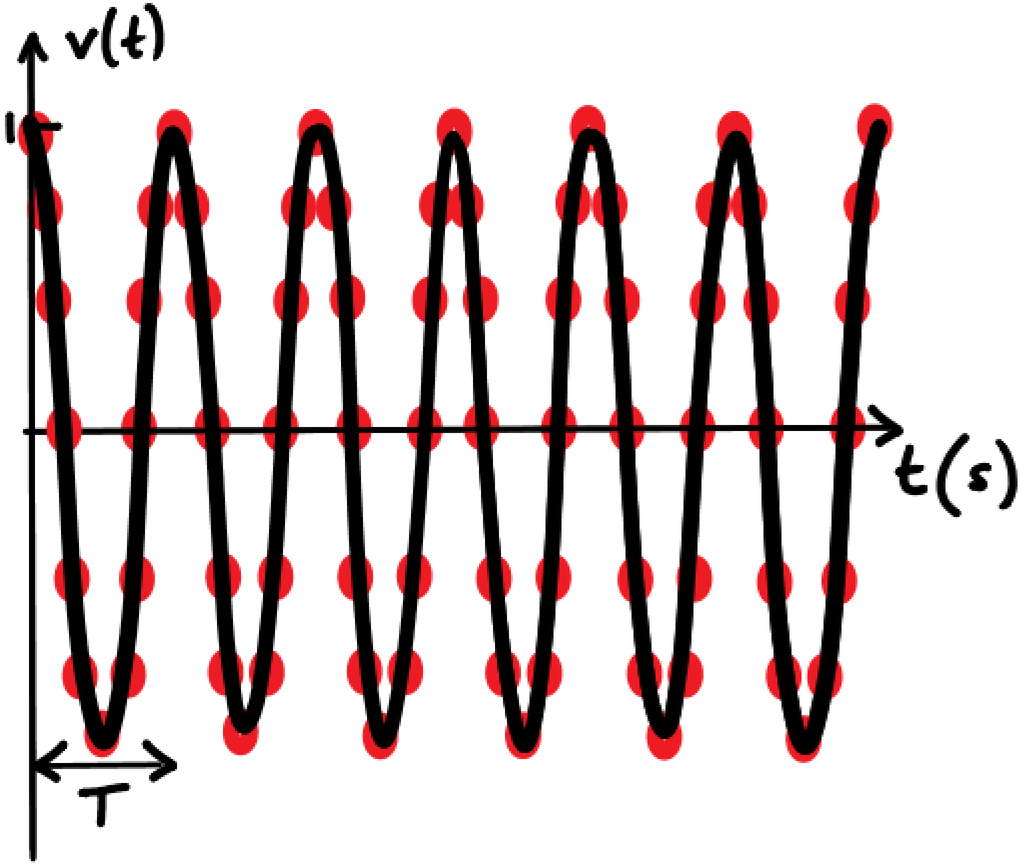
Higher rate sampling

Sampling at a high rate.

The signal v(t)=cos(2πft) is sampled uniformly with 12 sampling intervals within each signal period T.

The sampling interval $T_s = \frac{T}{12}$ and the sampling rate $f_s = 12f$.

The original signal x(t) can be recovered from the samples by connecting them together smoothly.



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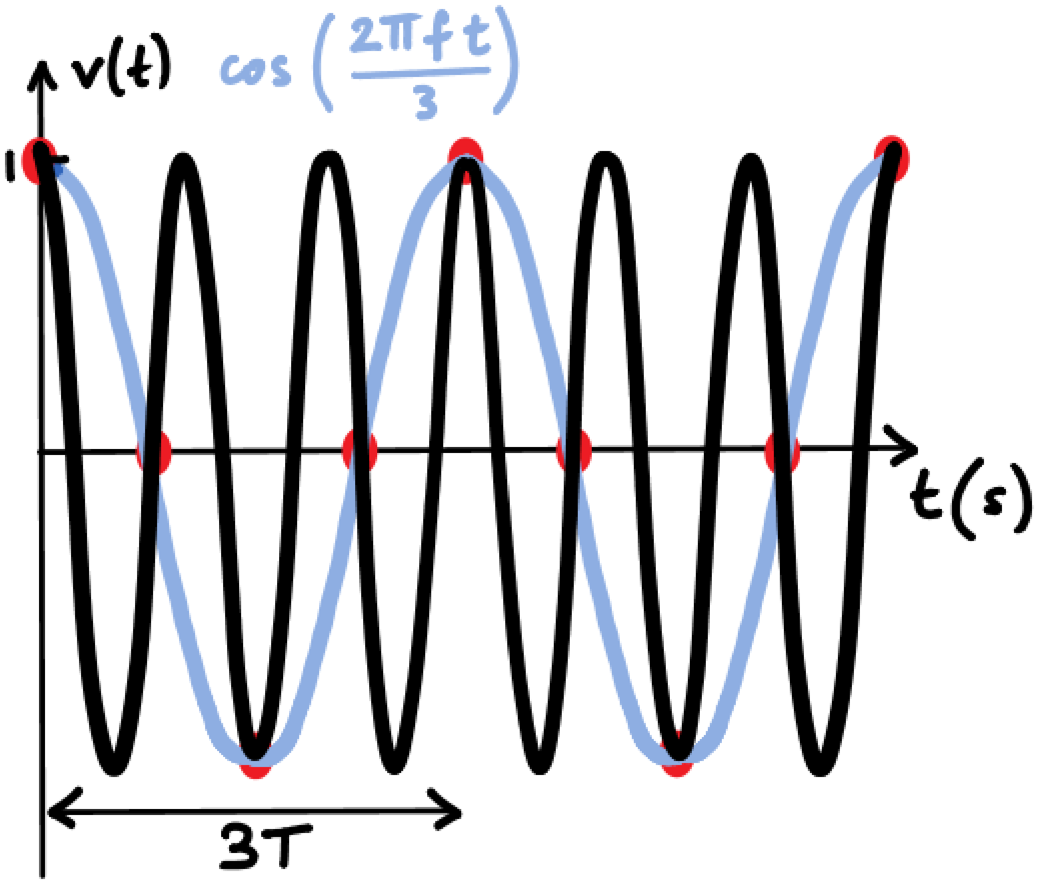
Lower rate sampling

In contrast, if a sinusoidal signal is sampled with a low sampling rate, the samples may be too infrequent to recover the original signal.



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Best sample rate

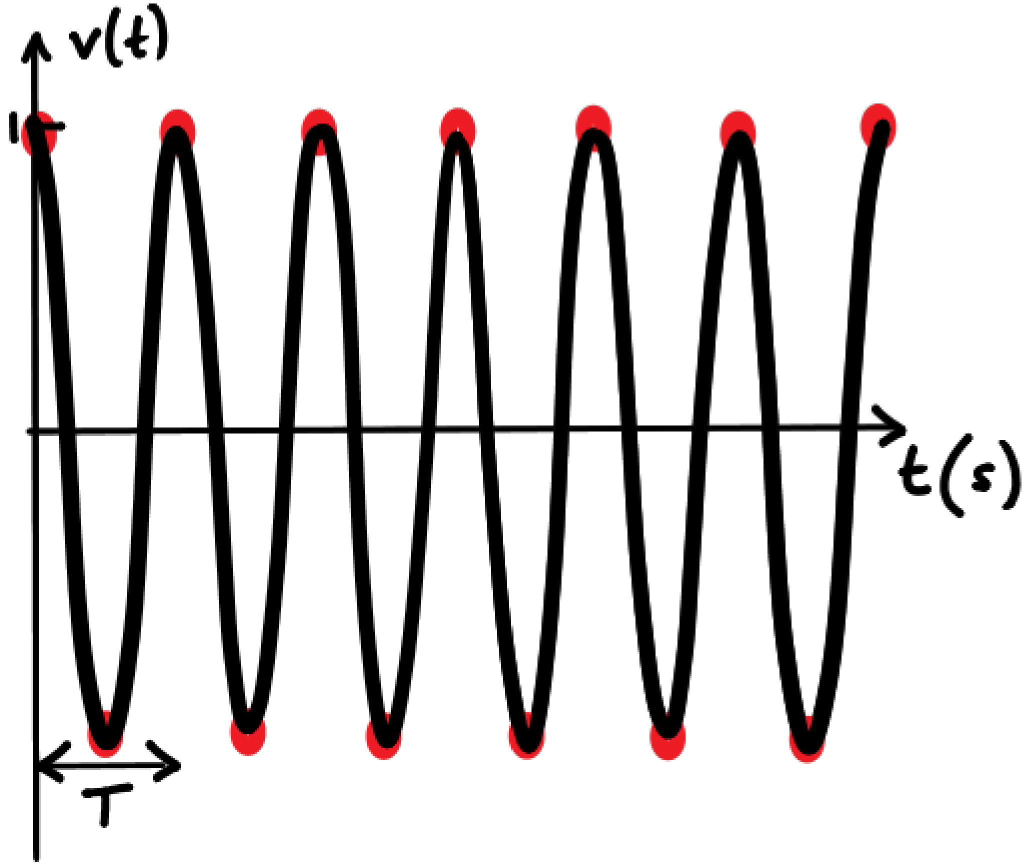
Sampling a cosine at $f_s = 2f$.

The signal v(t)=cos(2πft) is sampled uniformly with 2 sampling intervals within each signal period T.

sampling interval $T_s = \frac{T}{2}$ and the sampling rate $f_s = 2f$.

sample at every peak/trough of the sinusoid, there is no lower frequency sinusoid that fits these samples.

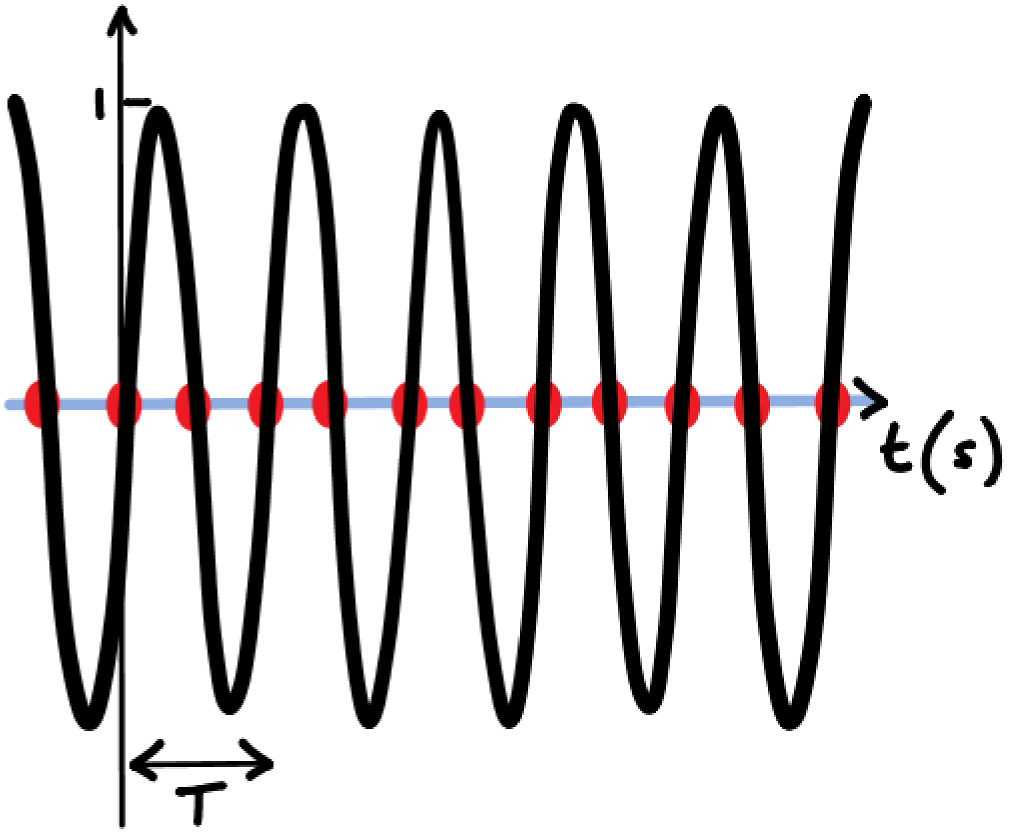
x(t) can be recovered exactly from the samples by ideal low pass filtering.



Worst case sample rate

The signal sin(2πft) is sampled uniformly with 2 sampling intervals within each signal period T.

Since all the samples are at the zero crossings, ideal low pass filtering produces a zero signal instead of recovering the sinusoid.



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So how do we decide sample rate

Nyquist-Shannon theorem

The Nyquist-Shannon sampling theorem

The sampling rate for exact recovery of a signal composed of a sum of sinusoids is larger than twice the maximum frequency of the signal.

This rate is called the Nyquist sampling rate $f_{Nyquist}$

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Terminology reminder

- Sampling is the process of recording an analog signal at regular discrete moments of time.
- The sampling rate f_s is the number of samples per second.
- The time interval between samples is called the sampling interval $T_s = \frac{1}{f_s}$.

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Theroem basics

- The sampling theorem:

  - **The sampling frequency must be greater than twice the bandwidth of the signal in order to recreate it perfectly**
    - $f_h < R/2$, where f_h is the frequency of the highest component of the signal, and R is the sampling rate

  - If you sample at too low a rate, **aliasing** or **foldover distortion** results

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Details

Sampling and Quantization

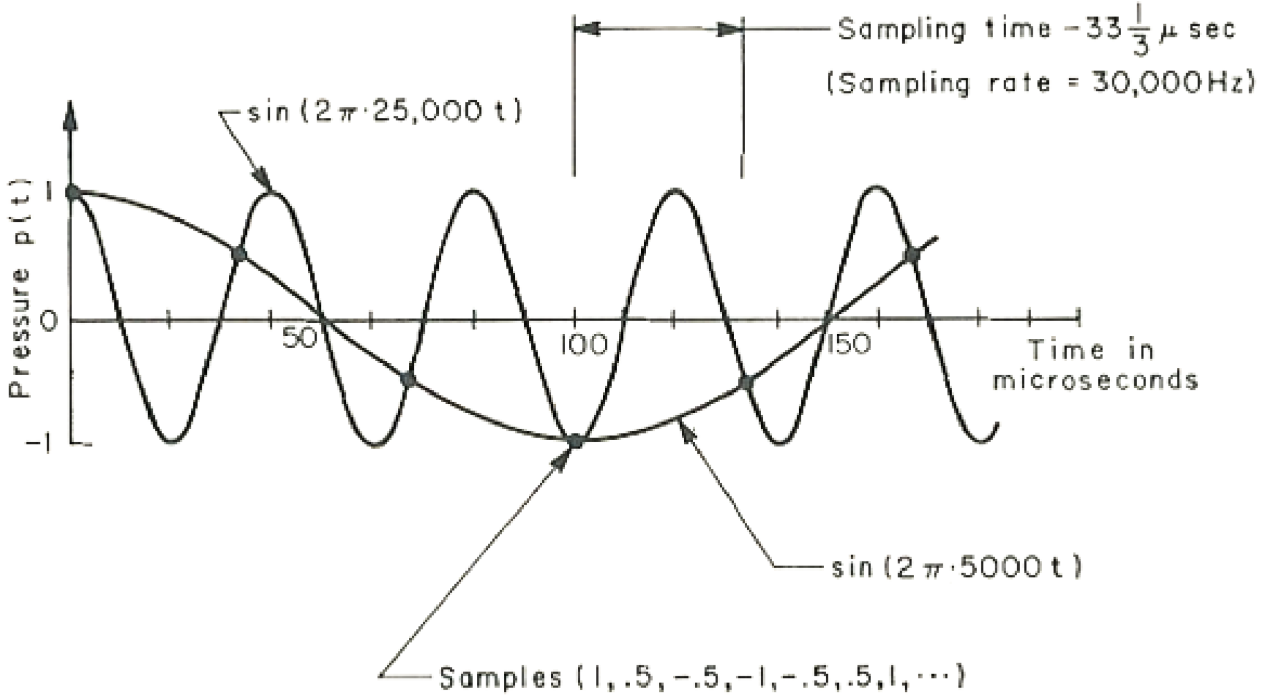

Fig. 5. Example of high-frequency (25,000 Hz) and foldover frequency (5000 Hz) resulting from low sampling rate (30,000 Hz).

Max Mathews: *The Technology of Computer Music*. MIT Press, 1969

Sampling and Quantization

- The frequency of the alias is calculated with:

$$F_a = \left| F - \frac{(k+1)R}{2} \right|, \qquad \frac{kR}{2} \leq F \leq \frac{(k+2)R}{2} \tag{1.1}$$

where

F_a	is the "apparent" frequency in Hz,
F	is the actual frequency in Hz,
R	is the sampling rate in Hz (samples per second), and
k	is any *odd* integer which satisfies the inequality.

Example

- The frequency of the alias is calculated with:

$$F_a = \left| F - \frac{(k+1)R}{2} \right|, \qquad \frac{kR}{2} \leq F \leq \frac{(k+2)R}{2} \qquad (1.1)$$

where

F_a is the "apparent" frequency in Hz,
F is the actual frequency in Hz,
R is the sampling rate in Hz (samples per second), and
k is any *odd* integer which satisfies the inequality.

- If F = 25000 Hz, and R = 30000 Hz

Example (cont'd)

- The frequency of the alias is calculated with:
  - If F = 25000 Hz, and R = 30000 Hz
  - $\frac{kR}{2} \leq F \leq \frac{(k+2)R}{2}$
  - $\frac{k*30000}{2} \leq 25000 \leq \frac{(k+2)*30000}{2}$
  - $k * 30000 \leq 50000 \leq (k+2) * 30000$
  - $k \leq 5/3 \leq (k+2)$
  - $k \leq 5/3 \leq (k+2)$

Example (cont'd)

- The frequency of the alias is calculated with:
  - If F = 25000 Hz, and R = 30000 Hz

  - $k \leq 5/3 \leq (k + 2)$
  - $k \leq 1.6666 \ldots \leq (k + 2)$
  - $1 \leq 1.6666 \ldots \leq 3$ when k =1

Example (cont'd)

- The frequency of the alias is calculated with:
  - If F = 25000 Hz, and R = 30000 Hz, then k = 1

  - $1 \leq 1.6666\ldots \leq 3$ when k =1
  - $F_a = \left|F - \frac{(k+1)R}{2}\right|$
  - $F_a = \left|30000 - \frac{(2)20000}{2}\right|$
  - $F_a = 5000$

Low-pass filtering

- To avoid aliasing, the signal is low-pass filtered before A/D conversion, eliminating any frequency components above R/2

More information

Some rates used

- Common audio sample rates:
  - CD: 44.1 kHz
    - Note: range of human hearing is 20 Hz to 20 kHz
- Pro audio: 48 kHz, 96 kHz, 192 kHz
- Speech codecs: 8000 Hz
- Apple lossless (maximum 384 kHz)
- Streaming music 44.1 kHz (some of this limit is contractual)

Digital form?

- The A/D converter quantizes the instantaneous amplitude of each sample
  - i.e. represents it using N-bit binary number
    - Normally a signed integer
  - The more bits the better, to improve the signal-to-noise ratio
    - E.g. 16 bits gives SNR of about 96 dB

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Commons sample bit sizes

- Common sample sizes:
  - CD/Stream: 16-bit
  - Pro audio (subscriber streams): 20-bit, 24-bit
  - Speech codecs: 8-bit, 12-bit

Onward to … spectral analysis.

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